英

秋季募集（令和６年度実施）

東北大学大学院工学研究科
量子エネルギー工学専攻入学試験

試験問題冊子
【専門科目　Specialized Subjects】

令和６年８月２８日(水)　10:00 ～11:30
Wednesday, August 28, 2024　10:00 ～11:30

Notice

1. Do not open this examination booklet until instructed to do so.

2. An examination booklet, answer sheets, draft sheets, and two subject selection forms are provided. Put your entrance examination ID on each of the answer sheets, the draft sheets, and the forms.

3. Indicate your selection on the subject selection forms and the answer sheets. Use two answer sheets for each subject.

4. At the end of the examination, double-check your entrance examination ID and the selected subject on the answer sheets. Put your answer sheets in numerical order on top of the other sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before the examiner's instruction.

流 体 力 学　FLUID DYNAMICS

Two complex velocity potentials $W_1(z)$ and $W_2(z)$ are given by

$$W_1(z) = -\frac{\Gamma}{2\pi i}\log_e(z-a)\,,\ \ W_2(z) = \frac{\Gamma}{2\pi i}\log_e(z+a).$$

Here z is a complex variable in the polar form given by $z = re^{i\theta}$, where r, θ and i are radial and circumferential coordinates, and the imaginary unit, respectively. Γ is a positive constant, and a is a real number constant.

(1) Regarding the flow given by the complex velocity potential $W(z) = W_1$ with $a = 0$, answer the following questions.

a) Obtain the velocity potential $\phi(r,\theta)$ and the stream function $\psi(r,\theta)$ of the flow field.

b) Obtain the radial component V_r and the circumferential component V_θ of the flow velocity.

c) Show that the flow is vortex-free except for the origin.

d) Obtain the pressure $p(r)$ at a distance r from the origin, when the pressure at a distance far enough from the origin is p_∞, the density of the fluid is ρ, and gravity is negligible.

(2) Regarding the flow given by the complex velocity potential $W(z) = W_1 + W_2$ with $a \neq 0$, answer the following questions.

a) Obtain the conjugate complex velocity of the flow field.

b) Obtain the velocity components in the real and imaginary axis directions on the real and imaginary axes.

電 磁 気 学　ELECTROMAGNETICS

Consider an electric dipole consisting of a point charge $+q$ at $(0, 0, d/2)$ and another point charge $-q$ at $(0, 0, -d/2)$, as shown in Fig. 1. The distance between the origin and point P is r, and the angle between the z-axis and the position vector of P is θ. The distances between the point charges with $\pm q$ and point P are r_1 and r_2, respectively. Let ε_0 be the permittivity of free space. Answer the following questions.

(1) Draw the schematic diagram of electric field lines around the electric dipole. Also indicate the direction of the electric field for each field line.

(2) Consider a sphere of radius r_0 and center point P. Obtain the integral of the normal component of $\boldsymbol{E}$ over the surface of the sphere, *S*, where $\boldsymbol{E}$ is the static electric field generated by the electric dipole. The normal vector pointing outward the sphere indicates the positive direction on *S*. Assume $r_2 > r_1$.

(3) Find the electrostatic potential V at point P as a function of r and θ under the condition that V is zero at infinity.

(4) When the distance r is far enough compared to the distance d, i.e., $r \gg d$, find the approximate expression of the electrostatic potential V obtained in question (3), $V_{approx}(r, \theta)$, by neglecting $(d/r)^n$ $(n \geq 2)$.

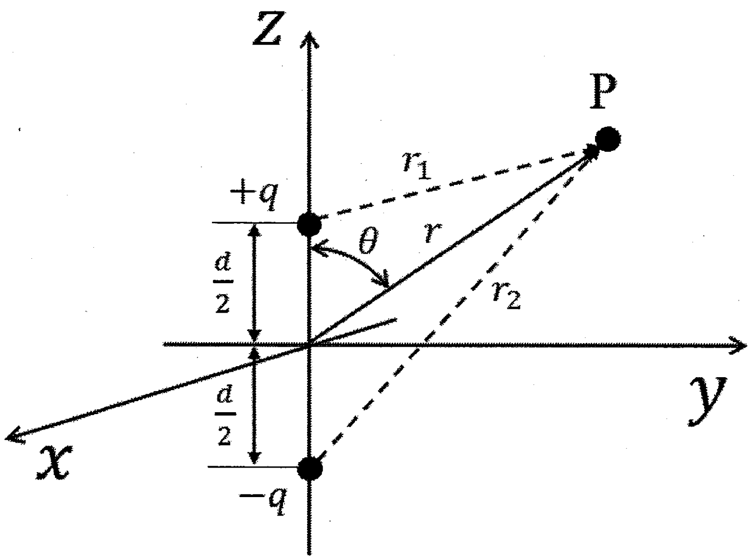


Fig. 1

量子力学 QUANTUM MECHANICS

When a particle with mass m is bound in the following one-dimensional potential

$$V(x) = \begin{cases} \infty & (x < 0) \\ 0 & (0 \leq x \leq d) \\ \infty & (d < x) \end{cases},$$

the wave function of the particle is expressed by $\psi(x,t) = u(x)T(t)$, where t is time and $d\ (> 0)$ is a constant. Planck's constant is given by h and $\hbar = h/(2\pi)$. Ignore the relativistic effects in the particle motion. Answer the following questions.

(1) Write the time-dependent Schrödinger equation for the particle.

(2) Find $u(x)$, $T(t)$ and the energies of the particle, E, in the region $0 \leq x \leq d$.

(3) Show $\partial|\psi(x,t)|^2/\partial t = 0$.

(4) Show that $\partial^2/\partial x^2$ is a Hermitian operator for the wave functions of the particle.

(5) Find the position uncertainty for the particle in the ground state by considering that the ground-state energy in question (2) is based on the uncertainty relationship between position and momentum. The uncertainty relationship is given by $\Delta x \Delta p \geq \hbar/2$, where Δx and Δp are the position and momentum uncertainties, respectively.

材料力学 STRENGTH OF MATERIALS

As shown in Fig. 1, there is a solid round bar of length L and diameter d. It is fixed to the wall at both ends, and a torsional moment M_t is applied at a distance x from the left end. The modulus of transverse elasticity of the material of the round bar is G. Answer the following questions.

(1) When $x = 0.3\,L$, find the torsional moment acting on each part of this round bar and draw it as a graph in the form of Fig. 2.

(2) When the loading point of the torsional moment M_t varies in the range $0.1\,L \leq x \leq 0.9\,L$, show the torsion angle at the loading point of the torsional moment M_t as a function of x.

(3) When the loading point of the torsional moment M_t varies in the range $0.1\,L \leq x \leq 0.9\,L$, show the maximum torsional moment that can be applied to the round bar as a function of x. Here, the allowable shear stress of the material of the round bar is given as τ_a.

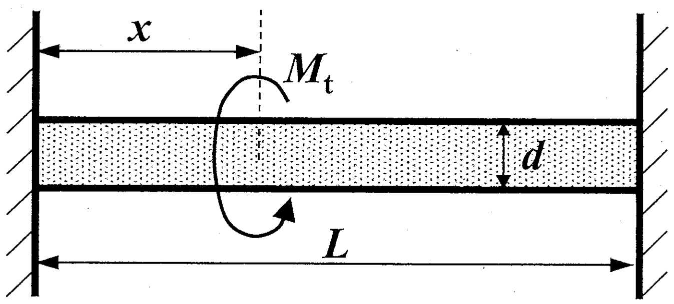


Fig. 1

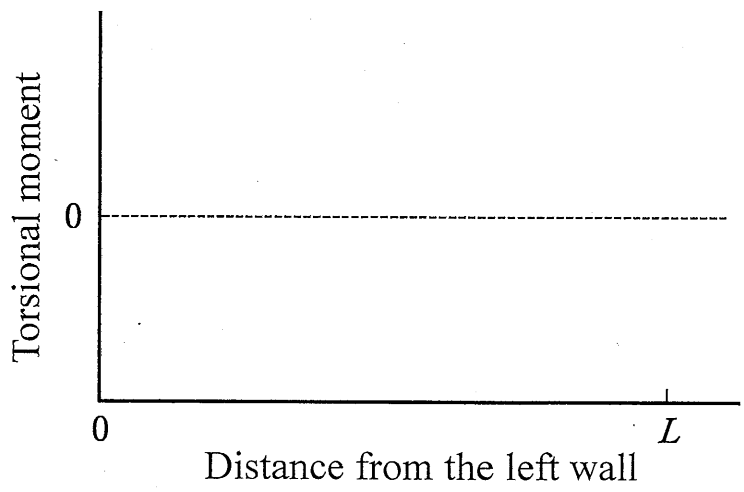


Fig. 2

機械材料学　MECHANICAL MATERIALS

Answer the following questions.

(1) Consider a body-centered cubic lattice which consists of elements A and B. Atoms of A are located at the corners of the unit cell, while an atom of B is located at the body-center position of the unit cell. Obtain the mole ratio of A to B in this material.

(2) Fig. 1 shows a schematic of the phase diagram of the Ni-Cu alloy forming a perfect solid solution. Draw schematic diagrams of the microstructures with chemical compositions of the contained phases formed at temperatures of a, b and c when a 35mass%Ni-65mass%Cu alloy is slowly cooled from 1350°C.

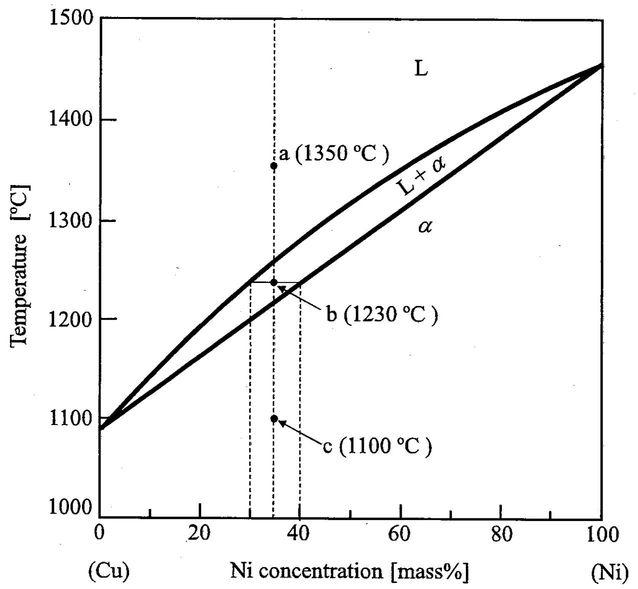


Fig. 1

(3) Explain the creep deformation of metals.

(4) The activation energy of carbon diffusion in iron is 8.0×10^4 J mol^{-1}. Give the temperature at which the diffusion coefficient of carbon in iron is half of its value at 700°C. If necessary, use the gas constant $R = 8.3$ J mol^{-1} K^{-1} and $\ln 2 = 0.7$.

(5) Fig. 2 shows the time temperature transformation diagram of eutectoid steel. Answer the following questions.

a) Explain why the “nose” temperature appears in Fig. 2.

b) Describe the difference in microstructures formed by the temperature histories of ① and ② in Fig.2.

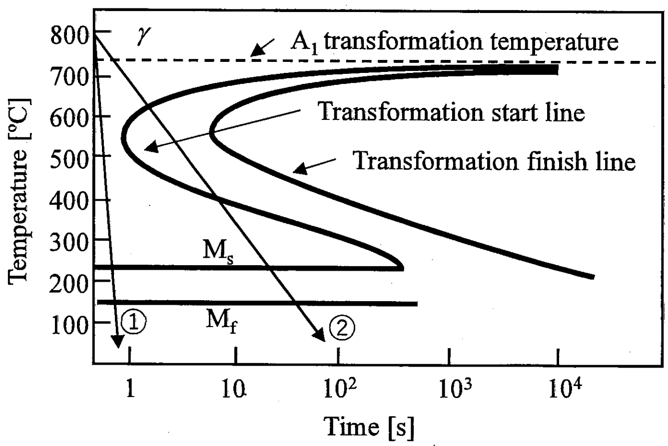
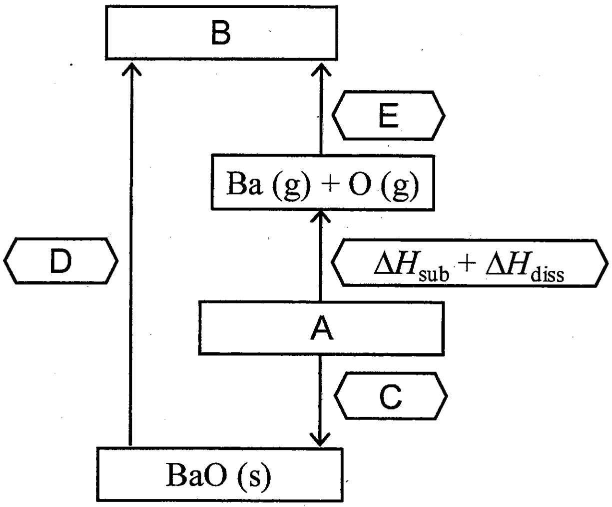


Fig. 2

化学基礎 CHEMISTRY BASICS

1. Answer the following questions concerning the Born-Haber cycle of BaO crystal in Fig. 1. Here, ΔH is reaction enthalpy.



Ba (s) → Ba (g)	ΔH_{sub} = +157 kJ mol^{-1}
Ba (g) → Ba^{2+} (g) + $2e^-$	ΔH_{ion} = +1462 kJ mol^{-1}
$1/2O_2$ (g) → O (g)	ΔH_{diss} = +249 kJ mol^{-1}
BaO (s) → Ba^{2+}(g) + O^{2-} (g)	ΔH_{lat} = +3130 kJ mol^{-1}
Ba (s) + $1/2O_2$ (g) → BaO (s)	ΔH_f = −560 kJ mol^{-1}
O (g) + $2e^-$ → O^{2-} (g)	ΔH_{elec}

Fig. 1

(1) Write the chemical formulae corresponding to A and B, and the enthalpy changes corresponding to C, D and E, in Fig. 1, respectively.

(2) Calculate ΔH_{elec} for the reaction of O (g) + $2e^-$ → O^{2-} (g).

2. Consider the following descriptions. If the underlined part is true, indicate "true". If the underlined part is false, indicate "false" and correct the underlined part.

(1) Electron affinity is the energy <u>released when an electron is added to a free atom</u>.

(2) When the standard electrode potentials of the half-reactions of "Cu^{2+} (aq) + $2e^-$ → Cu (s)" and "Zn^{2+} (aq) + $2e^-$ → Zn (s)" at 298 K are +0.34 V and −0.76 V, respectively, the standard electromotive force of the cell "Zn (s) | Zn^{2+} (aq) || Cu^{2+} (aq) | Cu (s)" is <u>+0.42 V</u>.

(3) Most hard acids and hard bases <u>are prone to</u> form covalent bonds.

(4) Among H_2O, $Fe(H_2O)_6^{3+}$ and $Fe(H_2O)_6^{2+}$, the strongest protonic acid is <u>$Fe(H_2O)_6^{2+}$</u>.

(5) The electronegativity of an atom depends on its ionization energy and electron affinity. The electronegativity <u>depends more on the electron affinity than on the ionization energy</u>.

(6) The angle between the C–H bonds of methane <u>is larger than</u> that between the O–H bonds of water.

放射化学　RADIOCHEMISTRY

Answer the following questions. In these questions, $\log_e 2 = 0.70$, and Avogadro's constant is 6.0×10^{23} mol^{-1}. The significant figure is two digits.

(1) Answer the following questions about neptunium.

a) A sample contains 0.70 MBq of ^{237}Np (half-life: 6.3×10^{13} sec. ≅ 2.0×10^6 years) and 6.0 MBq of ^{239}Np (half-life: 2.1×10^5 sec. ≅ 2.4 days). Show the total amount of Np in the sample in moles.

b) The decay mode of ^{239}Np is β^- decay. Show the nuclide produced by the decay of ^{239}Np.

c) Choose process(es) to produce ^{237}Np in a nuclear reactor from the following.

Write the answer using the symbol(s).

A. Fission of ^{235}U　　B. Fission of ^{239}Pu

C. β^- decay of the double neutron capture product of ^{235}U

D. α decay of ^{243}Am

(2) The atomic ratio of ^{232}Th to ^{224}Ra in an ore is $N(^{232}Th) : N(^{224}Ra) = 1.4 \times 10^{12} : 1.0$. Calculate the half-life of ^{224}Ra. Here, the half-life of ^{232}Th is 1.4×10^{10} years, and the sequential decays in the ore are in radioactive equilibrium.

(3) A mixed salt consisted of cesium chloride labeled by ^{134}Cs and strontium carbonate labeled by ^{90}Sr and ^{14}C. Using the mixed salt, an experiment was conducted in the order of a), b), and c).

a) When this salt was completely dissolved in dilute hydrochloric acid, a radioactive gas was generated. Show the radionuclide contained in the gas. Also, show the chemical reaction formula for this gas generation.

b) After the gas generation, a sodium sulfate solution was added to the mixed salt solution, so that a white precipitate was formed. Show the chemical reaction formula for this precipitate formation.

c) The white precipitate was collected and then left for 14 days, followed by an analysis. Show two types of radionuclides found in the white precipitate.

放 射 線 工 学
RADIATION ENGINEERING

1. A monochromatic alpha source and an ionization chamber were placed in a vacuum box. An energy spectrum of alpha rays emitted from the source was measured using the ionization chamber. As a result, a peak of 5 MeV was observed. The full width at half maximum (FWHM) and count rates of the peak were 0.3 MeV and 100 counts/second, respectively. The distance between the source and the window of the ionization chamber was 5 cm, and alpha rays entered to the window perpendicularly. The charge collection efficiency was constant, and the thickness of the window was negligibly thin. Answer the following questions <u>in each case</u>.

   (1) Gas in the ionization chamber is gradually vented. In the beginning, no change is observed, and then, the peak position begins to move to lower energy side in the energy spectrum. Explain the reason.

   (2) When air is introduced into the box gradually, a change in the energy spectrum is observed. Explain the changes of the peak position, the FWHM, and the count rate together with those reasons.

   (3) When the energy of alpha rays hypothetically becomes higher, explain the changes of the peak position and the count rate in the energy spectrum together with those reasons.

   (4) When the ionization chamber is replaced to a silicon semiconductor detector, the FWHM of the peak in the energy spectrum is decreased. Explain the reason.

2. Answer the following questions about the biological effects of radiations.

   (1) Explain the direct and indirect effects of radiations on DNA, respectively.

   (2) When mammal cultural cells were irradiated with alpha rays in the air, the Relative Biological Effectiveness (RBE) value became 3.0 at the cell survival rate of 0.01. Explain why this RBE value exceeded 1.0.

   (3) Mammal cultural cells were irradiated with X-rays under oxygen-free condition or oxygen-presence condition. Explain the difference between cell survival rates under the two conditions, together with those reasons.

   (4) Explain the protective effect of radiation protective agents.

原子炉物理学　REACTOR PHYSICS

1. Consider an infinitely large thermal reactor. The reactor is a homogeneous reactor consisting of a uniform mixture of fuel and moderator. The fuel consists of ^{238}U and ^{235}U. The atom densities of ^{238}U and ^{235}U in the fuel are N_{238} and N_{235}, respectively. The microscopic cross-sections of the absorption of thermal neutron of ^{238}U and ^{235}U are $\sigma_{a,238}$ and $\sigma_{a,235}$, respectively. The moderator's macroscopic cross-section of the absorption of thermal neutron is $\Sigma_{a,M}$. The fast fission factor, resonance escape probability, and average number of neutrons produced when the fuel absorbs one thermal neutron are given as ε, p, and η, respectively. Answer the following questions. Ignore the change in the composition of the fuel and moderator due to burn-up.
   (1) Obtain the fuel's macroscopic cross-section of the absorption of the thermal neutron.
   (2) Obtain the ratio of the number of thermal neutrons absorbed by the fuel and that by the moderator.
   (3) Show the three conditions that the reactor is subcritical, critical, and supercritical, respectively.

2. Fig. 1 shows changes in the power of a rector when a step change in reactivity is applied to the reactor. When a reactivity of $\Delta k/k = 0.5\%$ is applied, the power gradually increases from P_0 to P_1. In contrast, applying a reactivity of $\Delta k/k = 0.7\%$ causes an abrupt increase in the power followed by an abrupt decrease to reach a steady power of P_2. The fuel of the reactor is low-enriched uranium. Light water is used as the coolant and moderator of the reactor. Answer the following questions.
   (1) Explain the reason for the large difference in the changes in the power of the reactor.
   (2) Explain why the power of the reactor does not keep increasing but starts decreasing in the case that a reactivity of 0.7% is applied.
   (3) Qualitatively explain the response of the reactor when a reactivity larger than 0.7% is applied.

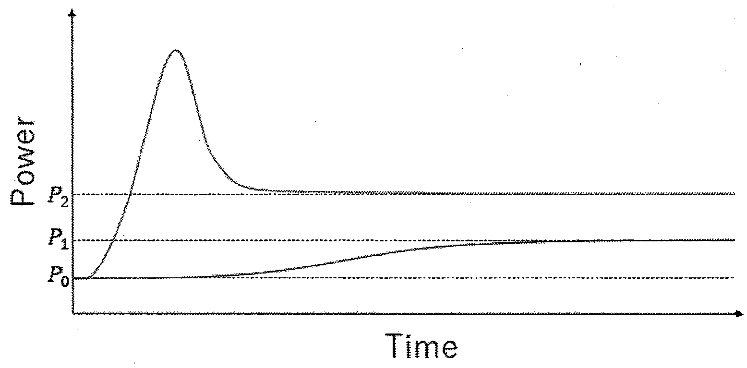


Fig. 1