

秋季募集（令和6年度実施）

東北大学大学院工学研究科
量子エネルギー工学専攻入学試験

試験問題冊子

数学B MATHEMATICS B

令和6年8月27日(火)

13:00 ~ 14:30

Tuesday, August 27, 2024

13:00 ~ 14:30

Notice

1. Do not open this examination booklet until instructed to do so.
2. An examination booklet, answer sheets, draft sheets are provided. Put your entrance examination ID-No. on each of the answer sheets and the draft sheets.
3. Answer all problems. Indicate the problem number on the answer sheets.
4. At the end of the examination, double-check your entrance examination ID-No. and the problem numbers on your answer sheets. Put your answer sheets in numerical order on top of the your draft sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

数学 B MATHEMATICS B

1. Find the general solutions of the following ordinary differential equations.

(1) $\frac{dy}{dx} = y(y - 1) \cos x$

(2) $x^2(xy + 1) \frac{dy}{dx} + (xy)^2 + xy + 1 = 0$ (Hint: put $u = xy$.)

(3) $\frac{d^2y}{dx^2} - 9 \frac{dy}{dx} + 20y = x^2 e^{3x}$

数学 B MATHEMATICS B

2. The Laplace transform of a function $f(t)$ is defined by

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^{\infty} f(t) e^{-st} dt.$$

Assume that the convergence condition of the Laplace transform is satisfied. Let a be a positive constant. Solve the following problems.

(1) Obtain the Laplace transform of the following functions.

a) $\frac{\sin at}{a}$

b) $e^{-at} \frac{\sin at}{a}$

(2) The convolution with respect to the function $f(t)$, denoted as $f * f$, is given by

$$f * f = \int_0^t f(t - \tau) f(\tau) d\tau.$$

Derive that the Laplace transform of $f * f$ is $\{F(s)\}^2$.

(3) Obtain the inverse Laplace transform of the following functions.

a) $\frac{1}{s^2 + 4s + 13}$

b) $\frac{1}{(s^2 + 4s + 13)^2}$ (Hint: use the result of problem (2).)