英

令和５年度 春季募集

東北大学大学院工学研究科
量子エネルギー工学専攻入学試験

試験問題冊子

数学B　MATHEMATICS B

令和６年２月２８日(水)　13:00 ～14:30
Wednesday, February 28, 2024　13:00 ～14:30

Notice

1. Do not open this examination booklet until instructed to do so.

2. An examination booklet, answer sheets, draft sheets are provided. Put your entrance examination ID-No. on each of the answer sheets and the draft sheets.

3. Answer all problems. Indicate the problem number on the answer sheets.

4. At the end of the examination, double-check your entrance examination ID-No. and the problem numbers on your answer sheets. Put your answer sheets in numerical order on top of the your draft sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

数 学 B　MATHEMATICS B

1. Find the general solutions of the following ordinary differential equations.

(1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = x + \sin 2x$

(2) $\frac{dy}{dx} - 2y = e^x$

(3) $\frac{dy}{dx} = \frac{2y^3 - x^3}{xy^2}$　(Hint : put $u = \frac{y}{x}$.)

2. Let $u(x,t)$ be the function satisfying the following partial differential equation,

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} - \lambda u \quad (0 < x < 1,\ t > 0),$$

with the boundary conditions,

$$u(x,0) = \sin \pi x,$$

and

$$u(0,t) = u(1,t) = 0,$$

where α and λ are positive constants. Solve the following problems.

(1) When $w(x,t) = u(x,t)e^{\lambda t}$, find the partial differential equation and boundary conditions satisfied by $w(x,t)$.

(2) Solve the partial differential equation given in problem (1) to find $w(x,t)$.

(3) Obtain $u(x,t)$, using the results of problems (1) and (2).