英

令和５年度 春季募集

東北大学大学院工学研究科
量子エネルギー工学専攻入学試験

試験問題冊子

数学A　MATHEMATICS A

令和６年２月２８日(水)　10:00 ～11:30
Wednesday, February 28, 2024　10:00 ～11:30

Notice

1. Do not open this examination booklet until instructed to do so.

2. An examination booklet, answer sheets, draft sheets are provided. Put your entrance examination ID-No. on each of the answer sheets and the draft sheets.

3. Answer all problems. Indicate the problem number on the answer sheets.

4. At the end of the examination, double-check your entrance examination ID-No. and the problem numbers on the answer sheets. Put your answer sheets in numerical order on your draft sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

数 学 A　MATHEMATICS A

1. Consider a curve on the xy plane given by

$$\begin{cases} x = a\cos^3\theta \\ y = a\sin^3\theta \end{cases} \quad (0 \le \theta \le 2\pi),$$

where θ is a parameter and a is a positive constant. Solve the following problems.

(1) Evaluate the length l of the curve.

(2) Evaluate the area S of the region enclosed by the curve.

2. A 3×3 matrix $\boldsymbol{A}$ is given by

$$\boldsymbol{A}=\begin{pmatrix} a & a-b & 0 \\ a-b & b & b-a \\ 0 & b-a & a \end{pmatrix} .$$

Solve the following problems. Here a and b are real constants, which satisfy $a>b$.

(1) Find the eigenvalues of $\boldsymbol{A}$ by solving its characteristic equation.

(2) Find three normalized eigenvectors of $\boldsymbol{A}$.

(3) Show three normalized eigenvectors obtained in problem (2) are orthogonal to each other.

(4) By using the results of problems (2) and (3), find the 3×3 diagonal matrix $\boldsymbol{D}$ and the 3×3 matrix $\boldsymbol{P}$ that satisfy $\boldsymbol{A}=\boldsymbol{PDP}^{-1}$.

3. In the three-dimensional Cartesian coordinate system (x, y, z), a vector field $\boldsymbol{A}$ is given by

$$\boldsymbol{A} = \left(\frac{x}{\sqrt{x^2+y^2+z^2}} + x^2, \frac{y-1}{\sqrt{x^2+y^2+z^2}} + 3x + 5y^2, \frac{z-1}{\sqrt{x^2+y^2+z^2}} + 3x + 2z^2 \right).$$

In addition, a region D is given by

$$D = \{ (x, y, z) |\ x^2 + (y-z)^2 \leq 1 \} .$$

Solve the following problems.

(1) Obtain $\nabla \times \boldsymbol{A}$ in the three-dimensional Cartesian coordinate system.

(2) Draw the region D.

(3) Obtain the position vector $\boldsymbol{r}$ of a point on a surface $y + z = 1$ and the unit normal vector $\boldsymbol{n}$ of the surface. Here the z component of $\boldsymbol{n}$ is non-negative.

(4) Let S be the plane of $y + z = 1$ in the region D. Draw the schematic shape of S and evaluate the area of S.

(5) Evaluate the line integral, $\int_C \boldsymbol{A} \cdot d\boldsymbol{r}$. Here C is a loop around the plane S obtained in problem (4).