英

令和５年度 秋季募集

東北大学大学院工学研究科
量子エネルギー工学専攻入学試験

試験問題冊子

数学B　MATHEMATICS B

令和５年８月２９日 (火)　　13:00 ～14:30
Tuesday, August 29, 2023　　13:00 ～14:30

Notice

1. Do not open this examination booklet until instructed to do so.

2. An examination booklet, answer sheets, draft sheets are provided. Put your entrance examination ID on each of the answer sheets and the draft sheets.

3. Answer all problems. Indicate the problem number on the answer sheets.

4. At the end of the examination, double-check your entrance examination ID and the problem numbers on your answer sheets. Put your answer sheets in numerical order on top of the other sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

数 学 B　MATHEMATICS B

1. Find the general solutions of the following ordinary differential equations.

(1) $(3x - y + 4)\dfrac{dy}{dx} - 2x + 3y + 1 = 0$

(2) $x^2\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = x^2 \quad (x > 0)$

(3) $e^y\dfrac{dy}{dx} + e^y = \sin x$

2. The Laplace transform of a function $f(t)$ is expressed by

$$\mathcal{L}[f(t)] = F(s) = \int_0^\infty f(t)e^{-st}dt.$$

Solve the following problems. Note that a is a constant, and n is a positive integer or zero in the problems. Assume that the condition of the convergence of the transform is satisfied.

(1) Show that $\mathcal{L}[e^{at}f(t)] = F(s-a)$.

(2) Find the Laplace transform of $f(t) = t^n$.

(3) Find the Laplace transform of $f(t) = e^{at}t^n$.

(4) Find the inverse Laplace transform of

$$F(s) = \frac{s}{(s-1)^3}.$$