英

令和５年度 秋季募集

東北大学大学院工学研究科
量子エネルギー工学専攻入学試験

試験問題冊子

数学Ａ　MATHEMATICS A

令和５年８月２９日(火)　10:00 ～11:30
Tuesday, August 29, 2023　10:00 ～11:30

Notice

1. Do not open this examination booklet until instructed to do so.

2. An examination booklet, answer sheets, draft sheets are provided. Put your entrance examination ID on each of the answer sheets and the draft sheets.

3. Answer all problems. Indicate the problem number on the answer sheets.

4. At the end of the examination, double-check your entrance examination ID and the problem numbers on the answer sheets. Put your answer sheets in numerical order on your draft sheet, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

数 学 A　MATHEMATICS A

1． Solve the following problems.

(1) Show the Taylor series of the following function about $x = 0$ up to the third order.
$f(x) = e^{2x}\cos x$

(2) Evaluate the following indefinite integral.

$$\int \frac{1}{\sqrt{a^2 + x^2}}\,dx \qquad (a > 0)$$

(3) Evaluate the following double integral.

$$\iint_D \sin(x+y)dxdy \ , \quad D = \{(x,y)|x \geq 0, \quad y \geq 0, \quad x + y \leq \pi\}$$

2. In the three-dimensional Cartesian coordinate system (x, y, z), a vector field $\boldsymbol{A}$ is given by

$$\boldsymbol{A} = \left(\frac{1}{x} + z^2\right)\boldsymbol{i} + \left(\frac{1}{y} + z^2\right)\boldsymbol{j} + \frac{1}{z}\boldsymbol{k},$$

where $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ are the fundamental vectors in the x, y, and z directions, respectively. In addition, regions D_1 and D_2 are given by

$$D_1 = \{ (x, y, z) \mid x^2 + y^2 + z^2 \leq 4\},$$

$$D_2 = \{ (x, y, z) \mid x^2 + y^2 \leq 2\}.$$

A region D_3 is defined as the region D_1 excluding the region D_2. Surfaces of the region D_3 corresponding to surfaces of the regions D_1 and D_2 are defined as curved surfaces S_1 and S_2, respectively. Here the three-dimensional polar and cylindrical coordinate systems are expressed as (r, θ, ϕ) and (ρ, ϕ, z), respectively. Solve the following problems.

(1) Obtain $\nabla \cdot \boldsymbol{A}$ and $\nabla \times \boldsymbol{A}$ in the three-dimensional Cartesian coordinate system.

(2) Draw the region D_3.

(3) Express a position vector of the curved surface S_1 with $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ and the three-dimensional polar coordinate variables of θ and ϕ. Then, find the ranges of θ and ϕ. In addition, express a position vector of the curved surface S_2 with $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ and the cylindrical coordinate variables of ϕ and z. In addition, find the ranges of ϕ and z.

(4) Obtain a summation of the areas of S_1 and S_2.

3. Two 3×3 matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are given by

$$\boldsymbol{A}=\begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{pmatrix} \quad \text{and} \quad \boldsymbol{B}=\begin{pmatrix} b & 1 & -1 \\ 1 & b & -1 \\ -1 & -1 & b \end{pmatrix}.$$

Solve the following problems. Here b is a real constant.

(1) Find the eigenvalues of $\boldsymbol{A}$ by solving its characteristic equation.

(2) Find three normalized eigenvectors of $\boldsymbol{A}$. If these eigenvectors are not orthogonal to each other, find the normalized orthogonal eigenvectors.

(3) Find the 3×3 diagonal matrix $\boldsymbol{D}$ and the 3×3 matrix $\boldsymbol{P}$ that satisfy $\boldsymbol{A}=\boldsymbol{P}\boldsymbol{D}\boldsymbol{P}^{-1}$.

(4) Find the eigenvalues and eigenvectors of $\boldsymbol{B}$.

(5) Find the 3×3 diagonal matrix $\boldsymbol{D}_B$ and the 3×3 matrix $\boldsymbol{P}_B$ that satisfy $\boldsymbol{B}=\boldsymbol{P}_B\boldsymbol{D}_B\boldsymbol{P}_B^{-1}$.