英

春季募集（令和 7 (2025) 年度実施）

東北大学大学院工学研究科
量子エネルギー工学専攻入学試験

試験問題冊子

数学A　MATHEMATICS A

2026年3月3日(火)　10:00 ～11:30
Tuesday, March 3, 2026　10:00 ～11:30

Notice

1. Do not open this examination booklet until instructed to do so.

2. An examination booklet, answer sheets, draft sheets are provided. Put your entrance examination ID-No. on each of the answer sheets and the draft sheets.

3. Answer all problems. Indicate the problem number on the answer sheets.

4. At the end of the examination, double-check your entrance examination ID-No. and the problem numbers on the answer sheets. Put your answer sheets in numerical order on your draft sheets, place them beside the examination booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

数 学 A　MATHEMATICS A

1. Consider a curve C on the xy plane given by

$$(x^2+y^2)^2 = a^2(x^2-y^2),$$

where a is a positive constant. Solve the following problems.

(1) By substituting $x = r\cos\theta$, $y = r\sin\theta$, derive the polar equation of the curve C in the form $r = f(\theta)$.

(2) Draw the curve C on the xy plane and calculate the area S of the region enclosed by the curve C.

(3) Find all the coordinates of the intersection points between the curve C and the circle $x^2+y^2 = \frac{a^2}{2}$.

2. In the three-dimensional Cartesian coordinate system, a region D and a plane S are given by

$$D = \{(x, y, z) | \, x^2 + y^2 + z^2 \leq 1 \},$$

$$S = \{(x, y, z) | x + y = 1\}.$$

Here, S_D is defined as a part of plane S inside the region D. Solve the following problems.

(1) Obtain the distance d between the origin and the plane S.

(2) Express the shape of S_D and obtain the area of S_D.

(3) A vector $\boldsymbol{A}$ is given by

$$\boldsymbol{A} = z\boldsymbol{i} + x\boldsymbol{j} + (-x + y)\boldsymbol{k},$$

where $\boldsymbol{i}, \boldsymbol{j}$, and $\boldsymbol{k}$ are the unit vectors along the x, y, and z directions, respectively. Obtain $\nabla \times \boldsymbol{A}$.

(4) Using the result of problem (3), evaluate the integral $\int_C \boldsymbol{A} \cdot d\boldsymbol{r}$, where C is the closed path along the boundary of S_D.

3. $f(x,y)$ is given by

$$f(x,y) = x^2 + 2(1-a)xy + 2y^2.$$

Here, a is a real number. Solve the following problems.

(1) Find a 2×2 matrix $\boldsymbol{A}$ when $f(x,y)$ can be transformed as follows. Here, non-diagonal elements of the matrix $\boldsymbol{A}$ are equal.

$$f(x,y) = \begin{pmatrix} x & y \end{pmatrix} \boldsymbol{A} \begin{pmatrix} x \\ y \end{pmatrix}$$

(2) Find the number of eigenvalues of the matrix $\boldsymbol{A}$ obtained in problem (1).

(3) Find the value of a when the matrix $\boldsymbol{A}$ obtained in problem (1) is diagonal. Find the matrix $\boldsymbol{A}$ for this value of a.

(4) Using the matrix $\boldsymbol{A}$ obtained in problem (3), find two eigenvalues of a matrix $\boldsymbol{D}$ when the matrix $\boldsymbol{A}$ satisfies $\boldsymbol{A} = \boldsymbol{P}^{-1}\boldsymbol{D}\boldsymbol{P}$. Here, matrices $\boldsymbol{D}$ and $\boldsymbol{P}$ are a 2×2 diagonal matrix and a 2×2 matrix, respectively.