

春季募集（令和 6（2024）年度実施）

東北大学大学院工学研究科  
量子エネルギー工学専攻入学試験

試験問題冊子

数学 B MATHEMATICS B

2025年3月4日(火)

13:00 ～14:30

Tuesday, March 4, 2025

13:00 ～14:30

### Notice

1. Do not open this examination booklet until instructed to do so.
2. An examination booklet, answer sheets, draft sheets are provided. Put your entrance examination ID-No. on each of the answer sheets and the draft sheets.
3. Answer all problems. Indicate the problem number on the answer sheets.
4. At the end of the examination, double-check your entrance examination ID-No. and the problem numbers on your answer sheets. Put your answer sheets in numerical order on top of the your draft sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

1. Find the general solutions of the following ordinary differential equations.

(1)  $(2y - x^2) \frac{dy}{dx} + \frac{1}{x} - 2xy = 0$

(2)  $\frac{1}{y^3} \frac{dy}{dx} - \frac{1}{xy^2} = 5x^2 + 3$

( Hint: put  $u = y^{-2}$ . )

(3)  $\frac{d^2y}{dx^2} - 6 \frac{dy}{dx} + 9y = e^{3x}$

2. The Laplace transform of a function  $f(t)$  is defined by

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^{\infty} f(t) e^{-st} dt.$$

A function  $u(t)$  is defined by

$$u(t) = \begin{cases} 0 & (t < 0) \\ 1 & (t \geq 0) \end{cases}.$$

When  $a$  is a positive constant, solve the following problems.

- (1) Show the following two equations,

$$\mathcal{L}\{u(t-a)f(t)\} = F(s) - \int_0^a f(t) e^{-st} dt,$$

$$\mathcal{L}\{u(t-a)f(t-a)\} = e^{-as}F(s).$$

- (2) When  $g(t)$  is given by  $g(t) = u(t-1) - u(t-2)$ , obtain  $G(s)$  which is the Laplace transform of the function  $g(t)$ .

- (3) When  $G(s)$  of problem (2) satisfies  $G(s) = (s^2 - 4s + 3)F(s)$ , obtain  $f(t)$ .

- (4) When  $g(t)$  is given by the following equation, obtain the Laplace transform of the function  $g(t)$ .

$$g(t) = u(t-1) - u(t-2) + u(t-3) - u(t-4) + \cdots = \sum_{n=1}^{\infty} (-1)^{n-1} u(t-n)$$