英

春季募集（令和６(2024)年度実施）

東北大学大学院工学研究科
量子エネルギー工学専攻入学試験

試験問題冊子

数学Ａ　MATHEMATICS A

2025年3月4日(火)　10:00 ～11:30

Tuesday, March 4, 2025　10:00 ～11:30

Notice

1. Do not open this examination booklet until instructed to do so.

2. An examination booklet, answer sheets, draft sheets are provided. Put your entrance examination ID-No. on each of the answer sheets and the draft sheets.

3. Answer all problems. Indicate the problem number on the answer sheets.

4. At the end of the examination, double-check your entrance examination ID-No. and the problem numbers on the answer sheets. Put your answer sheets in numerical order on your draft sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

数 学 A　MATHEMATICS A

1. Consider a curve on the xy plane given by

$$\begin{cases} x = a(1+\cos\theta)\cos\theta \\ y = a(1+\cos\theta)\sin\theta \end{cases} \quad (0 \le \theta \le 2\pi),$$

where θ is a parameter and a is a positive constant. Solve the following problems.

(1) Draw the curve on the xy plane.

(2) Evaluate the length l of the curve.

2. In the three-dimensional Cartesian coordinate system (x, y, z), a vector field $\boldsymbol{A}$ is given by

$$\boldsymbol{A} = \frac{x^2\sqrt{y^2+z^2}}{2}\,\boldsymbol{i} + \frac{z}{\sqrt{y^2+z^2}}\,\boldsymbol{j} + \frac{y}{\sqrt{y^2+z^2}}\,\boldsymbol{k},$$

where $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are the fundamental vectors in the x, y and z directions, respectively. Let S be the surface of the region D given by

$$D = \{\,(x, y, z) \mid 0 \le x \le 4,\ 1 \le y^2 + z^2 \le 4,\ y \ge 0\,\}.$$

Solve the following problems.

(1) Obtain $\nabla \cdot \boldsymbol{A}$ and $\nabla \times \boldsymbol{A}$ in the three-dimensional Cartesian coordinate system.

(2) Draw schematically the region D in the three-dimensional Cartesian coordinate system.

(3) Let $x = x$, $y = \rho\cos\phi$, $z = \rho\sin\phi$. Here $0 \le \phi \le 2\pi$. Find the ranges of ρ and ϕ in the region D, respectively.

(4) Obtain the Jacobian for the change of variables from the three-dimensional Cartesian coordinates (x, y, z) to the coordinates (x, ρ, ϕ) defined in problem (3).

(5) Evaluate the surface integral $\int_S \boldsymbol{A} \cdot \boldsymbol{n}\, dS$, where $\boldsymbol{n}$ is the outward unit normal vector of S.

3. The 2×2 matrix $\boldsymbol{A}$ is given by

$$\boldsymbol{A} = \begin{pmatrix} 1+a & 2a+1 \\ a-1 & -1 \end{pmatrix}.$$

Here, a is a real number larger than 0. Solve the following problems.

(1) Find eigenvalues of the matrix $\boldsymbol{A}$ and show that $\boldsymbol{A}$ has two different eigenvalues.

(2) Find two eigenvectors corresponding to the two different eigenvalues.

(3) When $a = 1$, find the matrices $\boldsymbol{D}$ and $\boldsymbol{P}$ that satisfy $\boldsymbol{A} = \boldsymbol{P}\boldsymbol{D}\boldsymbol{P}^{-1}$. Here, $\boldsymbol{D}$ and $\boldsymbol{P}$ are a 2×2 diagonal matrix and a 2×2 integer matrix (all components are integers), respectively.