英

秋季募集（令和７（2025）年度実施）

東北大学大学院工学研究科
量子エネルギー工学専攻入学試験

試験問題冊子
【専門科目　Specialized Subjects】

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2025年8月27日(水)　10:00 ～11:30
Wednesday, August 27, 2025　10:00 ～11:30

Notice

1. Do not open this examination booklet until instructed to do so.

2. An examination booklet, answer sheets, draft sheets, and two subject selection forms are provided. Put your entrance examination ID on each of the answer sheets, the draft sheets, and the forms.

3. Indicate your selection on the subject selection forms and the answer sheets. Use two answer sheets for each subject.

4. At the end of the examination, double-check your entrance examination ID and the selected subject on the answer sheets. Put your answer sheets in numerical order on top of the other sheets, place them beside the examination booklet, and wait for collection by an examiner. Do not leave your seat before the examiner's instruction.

流 体 力 学　FLUID DYNAMICS

As shown in Fig. 1, a liquid state Newtonian fluid with density ρ and viscosity coefficient μ flows on a wide and smooth plate which is inclined from a horizontal plane by an angle of θ. Regardless of location, the distance between the free surface of the fluid and the plate is constant at h. Let x-axis denote the direction along the inclined plane of the plate, and y-axis the direction perpendicular to the plate, where the origin of the y-axis is at the plate surface. The flow is steady, laminar and two-dimensional. The velocities of fluid in the x and y directions are $u = u(y)$ and $v = 0$, respectively. Consider a small rectangular parallelepiped control volume in the fluid. All faces of the control volume are parallel or perpendicular to the coordinate axes. Regarding the control volume, let the coordinates of the leftmost point, the lengths in the x and y directions be $(x,\ y)$, Δx and Δy, respectively, and let the normal and shear stresses, σ and τ, acting on each face be σ_x, τ_x, $\sigma_{x+\Delta x}$, $\tau_{x+\Delta x}$, σ_y, τ_y, $\sigma_{y+\Delta y}$ and $\tau_{y+\Delta y}$, respectively. Let $p = p(x,y)$ and g denote pressure in the fluid and gravitational acceleration, respectively. Assuming that ρ, μ, and g are constant, answer the following questions.

(1) Express σ_x, τ_x, σ_y and τ_y using the necessary symbols from p, u, y and μ.

(2) Express the balance of forces in the y direction using g, Δx, Δy, θ, ρ, σ_y, $\sigma_{y+\Delta y}$, τ_x and $\tau_{x+\Delta x}$.

(3) By using the result of question (2), obtain the momentum equation in the y direction using g, p, y, θ and ρ.

(4) By solving the equation obtained in question (3), obtain p. Assume that the pressure at the free surface is constant, $p = p_0$, regardless of location.

(5) By using the result of question (4), obtain the momentum equation in the x direction.

(6) By solving the equation obtained in question (5), obtain the profiles of τ_y and u, when $\tau_y = 0$ at the free surface.

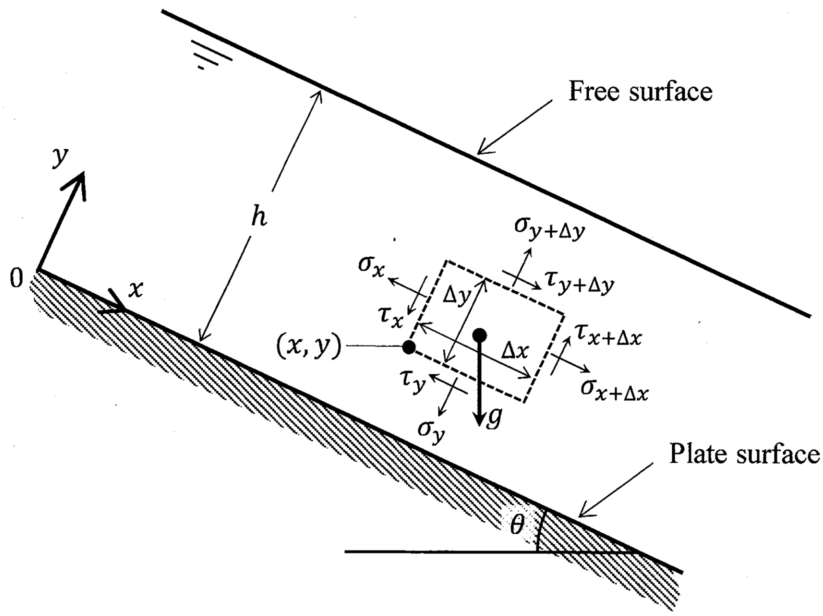

Fig. 1

電 磁 気 学　ELECTROMAGNETICS

As shown in Fig. 1, a square coil with 1 turn, sides of length a, resistance R is placed at a distance s from an infinitely long straight current I. The coil lies in the same plane as the current. Answer the following questions. Neglect the area of a cross section in the square coil conductor and the magnetic field created by the current induced in the square coil. Assume that the permeability is μ_0.

(1) Find the magnetic flux Φ that intersects the square coil.

(2) When the current I changes over time as indicated in the following equation, find the electromotive force ε generated in the square coil and the magnitude of the current i flowing in the square coil. Note that t is time, and λ and I_0 are positive constants.

$$I(t) = \begin{cases} (1-\lambda t)I_0 & (0 \le t \le \frac{1}{\lambda}) \\ 0 & (\frac{1}{\lambda} < t) \end{cases}$$

(3) Under the conditions in question (2), answer the direction of the current i with the reasons.

(4) Under the conditions in question (2), find the total charge Q passing through any cross section of the square coil conductor between $t = 0$ and $t = \infty$.

(5) Under the conditions in question (2), find the total magnitude and direction of the force on the square coil.

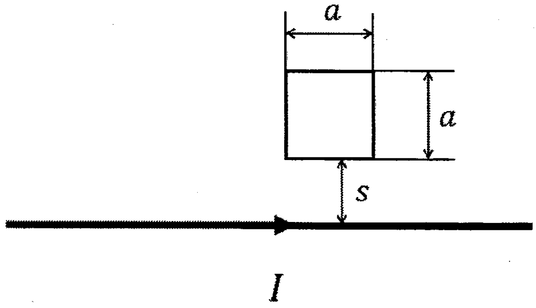

Fig. 1

量子力学 QUANTUM MECHANICS

Answer the following questions (1) ~ (3). Note that h is the Planck constant and $\hbar = h/(2\pi)$, and ignore the relativistic effects in the particle motion in questions (1) and (3), while the relativistic effects can be taken into account in question (2) if necessary.

(1) Let $P_r(r)dr$ be the probability that an orbital electron of a hydrogen atom exists between two spherical surfaces with radii r and $r+dr$ in the three-dimensional polar coordinate space (r, θ, ϕ). In addition, let n be principal quantum number and l be azimuthal quantum number. When $n = 1, l = 0$ and $n = 2, l = 1$, the trajectory probability density $P_r(r)$ is given as follows,

$$P_r(r) = \frac{4r^2}{{r_0}^3}\exp\left(-\frac{2r}{r_0}\right) \qquad (n = 1, l = 0),$$

$$P_r(r) = \frac{r^4}{24{r_0}^5}\exp\left(-\frac{r}{r_0}\right) \qquad (n = 2, l = 1),$$

where r_0 is the Bohr radius. Answer the following questions.

a) Find all r when $P_r(r)$ becomes its extremum for $n = 1, l = 0$ and $n = 2, l = 1$, respectively.

b) Let r_{10} and r_{21} be the radii with the highest probability of finding an orbital electron of a hydrogen atom for $n = 1, l = 0$ and $n = 2, l = 1$, respectively. Find the ratio of r_{21} to r_{10}.

(2) Consider a photon colliding with a free electron at rest (Fig. 1). The wave lengths of the incident and scattered photons are λ_0 and λ', respectively, while the scattering angle of the photon is φ. The momentum and recoil angle of the recoiled electron are p and θ, respectively.

a) Write momentum-conservation equations with respect to the incident direction and the direction perpendicular to it before and after the scattering.

b) Express the de Broglie wave length of the recoiled electron using λ_0, λ' and φ.

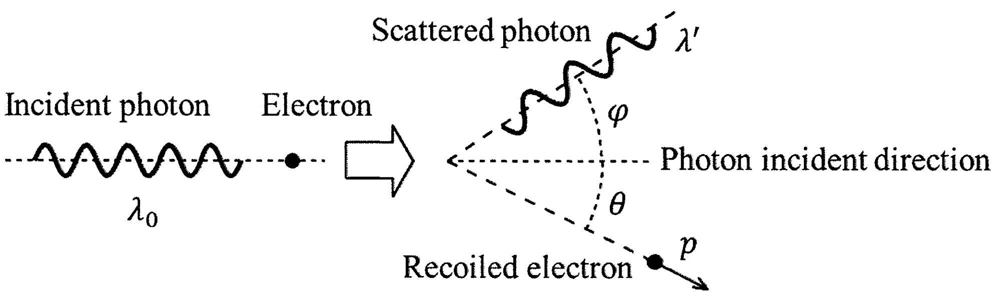

Fig. 1

(Question (3) is on the next page.)

(3) The wave functions $u_n(x)$ and energy values E_n of the eigenstates for a particle (mass m) in a one-dimensional harmonic potential $V(x) = m\omega^2 x^2/2$ are given by

$$u_n(x) = N_n H_n(\alpha x) e^{-\alpha^2 x^2/2} \text{ and } E_n = (n + 1/2)\hbar\omega \qquad (n = 0, 1, 2, \cdots),$$

respectively, where $N_n = \left(\alpha/\left(\sqrt{\pi}\, 2^n n!\right)\right)^{1/2}$ and $\alpha = \sqrt{(m\omega/\hbar)}$, and ω is the angular frequency. $H_n(\alpha x)$ is the nth Hermite polynomial, and expressed by

$$H_n(\alpha x) = (-1)^n e^{(\alpha x)^2} \frac{d^n e^{-(\alpha x)^2}}{d(\alpha x)^n},$$

where $H_0(\alpha x) = 1$. Answer the following questions.

a) When the one-dimensional potential $W(x)$ is

$$W(x) = \begin{cases} m\omega^2 x^2/2 & (0 \le x) \\ \infty & (x < 0) \end{cases},$$

find the ground-state wave function of the particle for $0 \le x$.

b) Find the energy eigenvalues E_s $(s = 0, 1, 2, \cdots)$ for $0 \le x$ in question a).

材料力学 STRENGTH OF MATERIALS

Consider a solid circular shaft and a hollow circular shaft, each made of a different brittle material. The solid shaft has a length L, diameter d_1, shear modulus G_1, and tensile strength σ_{F1}, while the hollow shaft has a length L, inner diameter d_1, outer diameter d_2, shear modulus G_2, and tensile strength σ_{F2}. Answer the following questions.

(1) The left end of the hollow shaft is fixed to a rigid wall, and a torsional moment M_t is applied to the right end. Determine the angle of twist and the maximum shear stress in the hollow shaft.

(2) As shown in Fig. 1, the solid shaft is inserted into the hollow shaft to form a combined shaft. The left end of the combined shaft is fixed to a rigid wall, and a rigid circular plate is bonded to its right end. Subsequently, the circular plate is rotated by an angle ϕ. Determine the torsional moment applied to the circular plate.

(3) Express the condition under which the solid shaft fractures before the hollow shaft, using d_1, d_2, G_1, G_2, σ_{F1}, and σ_{F2}, when ϕ is gradually increased in question (2). Also, predict the angle of a crack formed by the torsion relative to the axis of the shaft and explain the reason for the prediction.

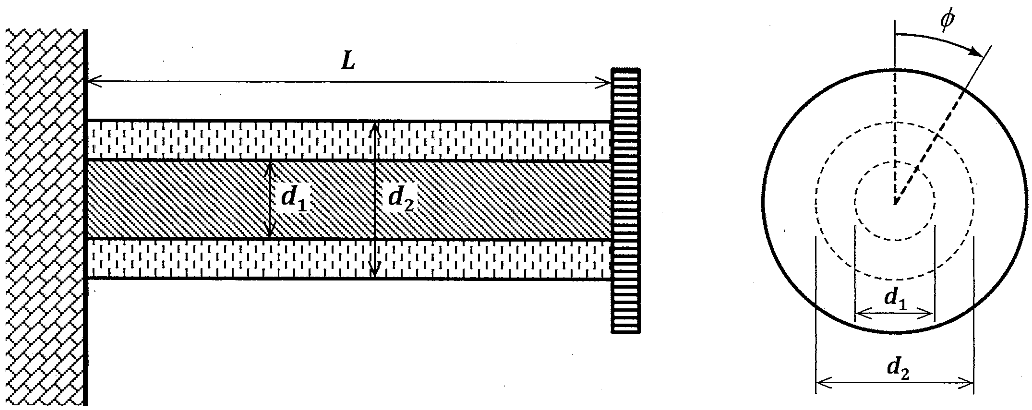

Fig. 1

放射線基礎　RADIATION BASICS

1. Answer the following questions. In these questions, $\log_e 2 = 0.70$, and Avogadro's constant is 6.0×10^{23} $\mathrm{mol^{-1}}$.

   (1) 1.0 g of ^{235}U (the 92nd element, α decay, half-life: 7.0×10^8 years) and 2.4 mg of ^{237}U (β^- decay, half-life: 7.0 days) are present in a test tube.

   a) Answer the atomic number, mass number, and decay mode of the daughter nuclide of ^{237}U.

   b) Calculate the specific activity (Bq/g) of uranium in the test tube. The significant figure is two digits.

   (2) A rare earth ore was dissolved into 2.0 L of aqueous solution. This aqueous solution was contacted with 1.0 L of an organic solvent containing an extractant for neodymium and samarium solvent extraction. After the extraction, the concentrations of ^{143}Nd and ^{147}Sm in the organic phase were m_1 mol/L and n_1 mol/L, respectively. In this extraction, neodymium and samarium distribution ratios were D(Nd) = 8.0, D(Sm) = 1.0×10^1, respectively.

   a) Show the amount of substances (mol) of ^{143}Nd and ^{147}Sm in the rare earth ore before the dissolution.

   b) ^{147}Sm is a natural radionuclide. ^{147}Sm decays to ^{143}Nd (stable isotope) by α decay with a half-life of 1.0×10^{11} y. When the rare earth ore was formed, m_0 mol of ^{143}Nd was contained in it. Using the given conditions, derive a formula that indicates the elapsed time t (y) since the ore was formed.

2. A radioactive nuclide Y (rest mass: m_Y) is produced from stable nuclide X (atomic number: Z, mass number: M, rest mass: m_X) by the following nuclear reaction.

$$\mathrm{X} + \mathrm{n} \rightarrow \mathrm{Y} + \gamma .$$

Here, n and γ show a neutron and a photon of gamma ray, respectively. The Q value of the nuclear reaction is q. Y decays only by β^- decay, and the decay constant is λ. When atomic nuclei of X (number of nuclei: N_X) were irradiated with monoenergetic neutrons at a constant flux f, the number of generated nuclei of Y was N after time t has elapsed since the start of irradiation. The microscopic nuclear reaction cross section is σ, and N_X is regarded as a constant. When the speed of light in a vacuum is c, answer the following questions using the necessary symbols defined in the text.

   (1) Answer the atomic number and mass number of Y and those of a nuclide generated by decay of Y, respectively.

   (2) Express the rest mass of neutron.

   (3) Show the equation of dN/dt, the change amount of N per unit time.

   (4) Beam irradiation stopped at $t = T$, and then the activity of Y was immediately measured. Answer the time T_{half} ($< T$) at which the activity was half of the measured value, based on the result of (3).