英

秋季募集（令和 7 (2025) 年度実施）

東北大学大学院工学研究科
量子エネルギー工学専攻入学試験

試験問題冊子

数学B　MATHEMATICS B

2025年8月26日(火)　13:00 ～14:30

Tuesday, August 26, 2025　13:00 ～14:30

Notice

1. Do not open this examination booklet until instructed to do so.

2. An examination booklet, answer sheets, draft sheets are provided. Put your entrance examination ID-No. on each of the answer sheets and the draft sheets.

3. Answer all problems. Indicate the problem number on the answer sheets.

4. At the end of the examination, double-check your entrance examination ID-No. and the problem numbers on your answer sheets. Put your answer sheets in numerical order on top of the your draft sheets, place them beside the examination booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

数 学 B　MATHEMATICS B

1. Find the general solutions of the following ordinary differential equations.

(1) $\left(y^2 - \dfrac{3x}{y} + 2\right)\dfrac{dy}{dx} + \left(\dfrac{x^3}{y} + \dfrac{2x}{y} - 3\right) = 0$

(2) $\dfrac{d^2y}{dx^2} + \dfrac{1}{4}y = \sin\dfrac{x}{2}$

(3) $x\exp(x - 2y) + y\exp(-x + y)\dfrac{dy}{dx} = 0$

2. The Laplace transform of a function $y(t)$ is defined by

$$\mathcal{L}\{y(t)\} = Y(s) = \int_0^\infty y(t)\, e^{-st} dt.$$

The functions $u(t)$, $p_a(t)$ and $\delta(t)$ are defined by

$$u(t) = \begin{cases} 0 & (t < 0) \\ 1 & (t \geq 0) \end{cases},$$

$$p_a(t) = u(t) - u(t-a),$$

$$\delta(t) = \lim_{a \to 0} \frac{p_a(t)}{a},$$

where a is a positive constant. Solve the following problems.

(1) Obtain the Laplace transform of $p_a(t)$.

(2) When integration and limit operations are commutative, show $\mathcal{L}\{\delta(t)\} = 1$.

(3) Obtain the Laplace transform of $\delta(t-a)$.

(4) When $y(t)$ satisfies the following differential equation, express $Y(s)$ using $U(s)$ by applying the Laplace transform to both sides. Here, $U(s)$ denotes the Laplace transform of $u(t)$, and let $y(0)$, $y'(0)$, $u(0)$, and $u'(0)$ be 0.

$$y'' + 4y' + 13y = u'' - 2u' + u$$

(5) In problem (4), obtain $y(t)$ when $u(t) = \delta(t-a)$.