英

秋季募集（令和 7 (2025) 年度実施）

東北大学大学院工学研究科
量子エネルギー工学専攻入学試験

試験問題冊子

数学A　MATHEMATICS A

2025年8月26日(火)　10:00 ～11:30
Tuesday, August 26, 2025　10:00 ～11:30

Notice

1. Do not open this examination booklet until instructed to do so.

2. An examination booklet, answer sheets, draft sheets are provided. Put your entrance examination ID-No. on each of the answer sheets and the draft sheets.

3. Answer all problems. Indicate the problem number on the answer sheets.

4. At the end of the examination, double-check your entrance examination ID-No. and the problem numbers on the answer sheets. Put your answer sheets in numerical order on your draft sheets, place them beside the examination booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

数 学 A　MATHEMATICS A

1．Solve the following problems.

(1) Evaluate the following indefinite integral.

$$\int \frac{1}{2}\cos\sqrt{x}\,dx$$

(2) Draw region D on the xy plane and evaluate the following multiple integrals.

$$\iint_D xy\,dxdy \quad , \quad D=\{(x,y)|x\geq 0, \quad y\geq 0, \quad xy\geq\sqrt{3}, \quad x^2+y^2\leq 4\}$$

2. In the three-dimensional Cartesian coordinate system (x, y, z), a vector field $\boldsymbol{A}$ is given by

$$\boldsymbol{A} = (x + y)\boldsymbol{i} + (2x - z)\boldsymbol{j} + (y + z)\boldsymbol{k},$$

where $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$ are the fundamental vectors in the x, y and z directions, respectively. A triangle S is defined by the line segments of three points P, Q and R that are the intersections of the plane $3x + 2y + z = 6$ and the axes of the three-dimensional Cartesian coordinate system. The side of S that does not face the origin is the outward side of S. Solve the following problems.

(1) Obtain $\nabla \cdot \boldsymbol{A}, \nabla \times \boldsymbol{A}$ in the three-dimensional Cartesian coordinate system.

(2) Obtain the distance between the triangle S and the origin.

(3) Obtain the outward unit normal vector $\boldsymbol{n}$ and the surface area s of the triangle S.

(4) Evaluate the line integral $\int_C \boldsymbol{A} \cdot d\boldsymbol{r}$, where C is the path PQ + QR + RP.

3. The 2×2 matrix $\boldsymbol{A}$ and $f(x,y)$ are given by

$$\boldsymbol{A} = \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}, \quad f(x,y) = 3x^2 - 2xy + 3y^2 .$$

Solve the following problems.

(1) Find two eigenvalues and normalized eigenvectors of the matrix $\boldsymbol{A}$.

(2) Using the results obtained in problem (1), find the matrices $\boldsymbol{D}$ and $\boldsymbol{P}$ that satisfy $\boldsymbol{A} = \boldsymbol{PDP}^{-1}$. Here, $\boldsymbol{D}$ and $\boldsymbol{P}$ are a 2×2 diagonal matrix and a 2×2 matrix, respectively. The diagonal components of $\boldsymbol{P}$ are positive.

(3) Find the matrix $\boldsymbol{Q}$ when $f(x,y)$ can be transformed as follows.

$$f(x,y) = \begin{pmatrix} x & y \end{pmatrix} \boldsymbol{Q} \begin{pmatrix} x \\ y \end{pmatrix} .$$

(4) Using the matrix $\boldsymbol{P}$ obtained in problem (2), x and y are transformed as follows. Find the lengths of major and minor axes of ellipse $f(x,y) = 1$ based on the results obtained in problems (2) and (3).

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \boldsymbol{P} \begin{pmatrix} x \\ y \end{pmatrix} .$$