Reference Problems - STRENGTH OF MATERIALS

1. There are three columns whose cross-sectional area is A_0 and whose natural length at temperature T_0 is L_0. These three columns are bonded to the rigid plates at the top and bottom ends, as shown in Fig. 1. The column located in the center of the rigid plate has a longitudinal modulus of elasticity of E_1 and a linear expansion coefficient of α_1, while the two columns located at distance C on both sides of the central column have a longitudinal modulus of elasticity of E_2 and a linear expansion coefficient of α_2. Here, $0 < E_1 < E_2$ and $0 < \alpha_1 < \alpha_2$. Ignore the self-weight of the columns and rigid plates. Answer the following questions.

   (1) At temperature T_0, an object of weight W_0 is placed on the center of the upper rigid plate, as shown in Fig. 2. Find the strain in the columns. Also, find the stress acting on each column.

   (2) After the object of weight W_0 is removed from the state in the question (1), the temperature is increased by ΔT from T_0. Find the strain in the columns. Also, find the stress acting on each column.

   (3) The object of weight W_0 is placed on the center of the upper rigid plate as in the question (1), and the temperature is increased by ΔT from T_0. Find the stress acting on each column.

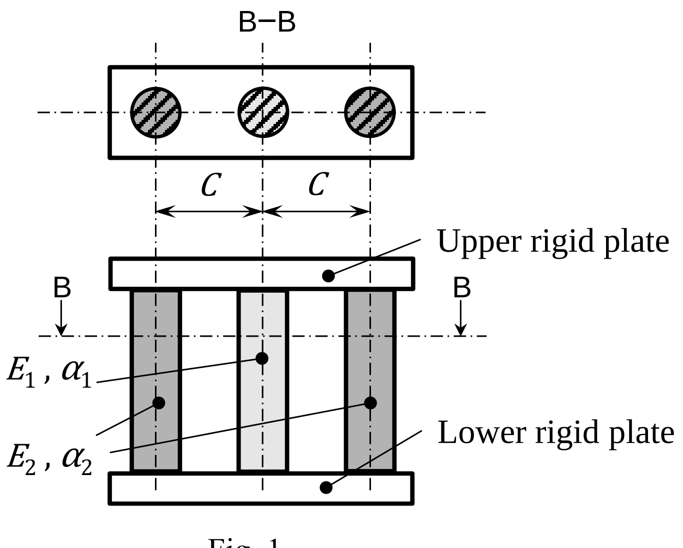

Fig. 1

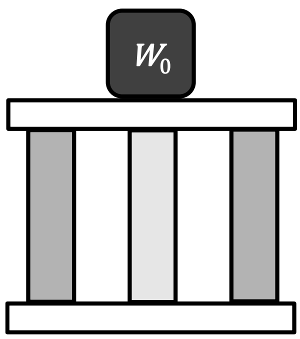

Fig. 2

2. Answer the following questions.

(1) The plane stresses (σ_x=200 MPa, σ_y=0 MPa, τ_{xy}=100 MPa) are given as shown in Fig. 3. Draw the Mohr's stress circle corresponding to this stress state. Find the magnitude and direction of the maximum and minimum principal stresses.

(2) The yield strength of a homogeneous isotropic material was evaluated by a uniaxial tensile test, and the material yielded when the tensile stress reached 250 MPa. Determine whether or not this material will yield when subjected to the plane stress as shown in Fig. 3, and the reason should also be given. Here, the yield will occur when the maximum shear stress reaches the material-specific limit value.

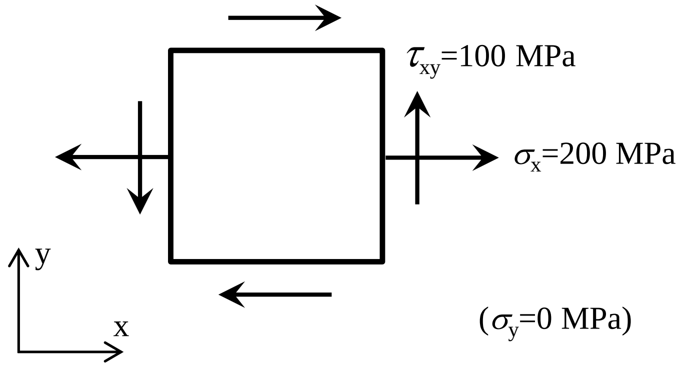

Fig. 3

3. Consider a stepped solid circular shaft shown in Fig. 4. The diameter and length of section AB are d_1 and $L/3$, and those of section BD are d_2 and $2L/3$, respectively. d_1 is smaller than d_2. The shaft is fixed at the both ends, A and D, to vertical rigid walls. The shear modulus of the shaft material is G. As shown in Fig. 1, the shaft is twisted by the torsional moment M_t at position C, midpoint of section BD. Answer the following questions. Describe also the processes of deriving the answers. Ignore stress concentration.

(1) Find the torsional moments developed at the ends of the shaft, position A and

position D, respectively.

(2) Find the angle of twist ϕ_c at position C.

(3) Determine the ratio of torsional stress acting on section AB and torsional stress acting on section CD.

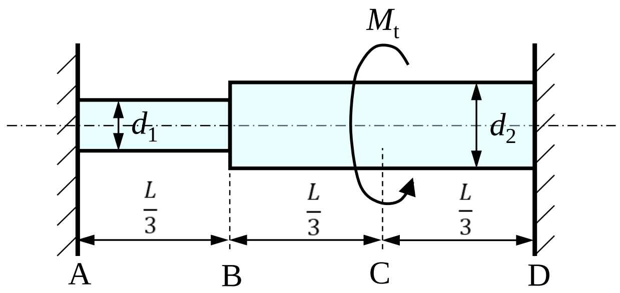

Fig. 4

4. A $3L$-long beam is supported at two points A and B at a distance L from both ends of the beam, as shown in Fig. 5. A concentrated load W_1 is applied at the left end of the beam and a concentrated load W_2 is applied at the right end, where $2W_2 > W_1 > W_2$. Answer the following questions. Neglect the weight of the beam. Describe also the processes of deriving the answers.

(1) Determine the reaction forces acting at position A and position B, respectively.

(2) Draw the shear force diagram.

(3) Draw the bending moment diagram.

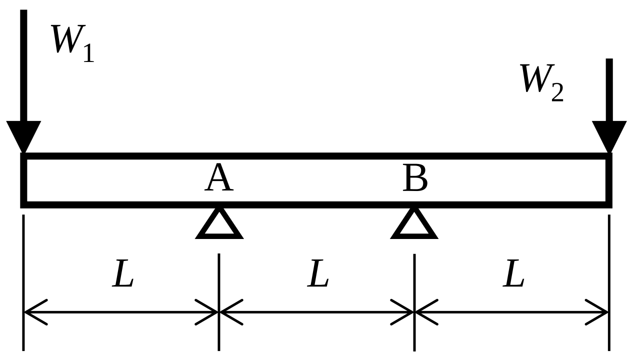

Fig. 5

5. There is a solid circular shaft with diameter, d, and total length, $4L$, both ends of which are fixed to the wall. The shaft is made of a brittle material and its shear modulus of elasticity is G. A torsional moment, M_t, is acting on the shaft at the position L from its left end, and a torsional moment twice as large (i.e., $2M_t$) is acting in the same direction on the shaft at the position L from its right end, Figure 6. Here, M_t is a variable. It is known that if magnitude of M_t is gradually increased and when M_t reaches M_{t0}, the shaft fails with exhibiting fracture surface characteristics of brittle materials. Answer the following questions.

(1) Find the torsional moment of each part of the shaft and draw a graph in the form of Figure 7.

(2) Answer the direction in which the fracture surface is expected to be formed, together with the reason.

(3) Assuming that failure occurs when the maximum principal stress of the shaft reaches the tensile strength of the material, σ_{TS}, find the relationship between σ_{TS} and M_{t0}.

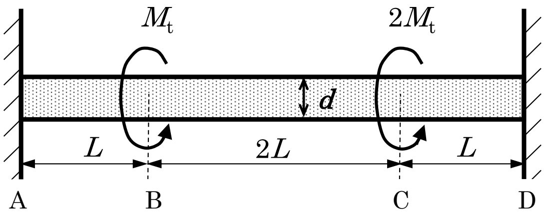

Fig. 6

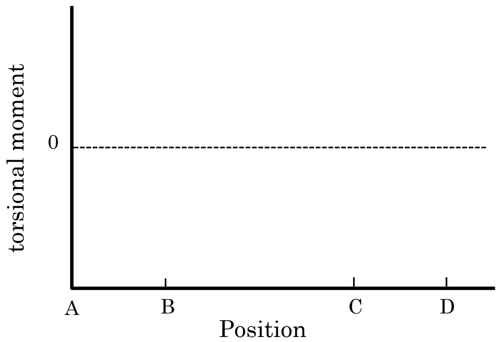

Fig. 7