英

平成 29 年度 秋季募集
(平成 29 年 10 月・平成 30 年 4 月入学)
東北大学大学院機械・知能系入学試験

試験問題冊子

数学A MATHEMATICS A

平成 29 年 8 月 29 日(火)
Tuesday, August 29, 2017 9:30 − 11:00

Notice

1. Do not open this test booklet until instructed to do so.
2. A test booklet, answer sheets, draft sheets, and a selected-problems form are provided. Put your examinee number on each of the answer sheets, the draft sheets, and the form.
3. Select three of the four problems and answer them. Indicate your selection on the selected-problems form. Use two answer sheets for each problem.
4. At the end of the examination, double-check your examinee number and the problem numbers on the answer sheets. Put your answer sheets in numerical order on top of the other sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

数 学 A　MATHEMATICS A

1. Solve the following problems.

(1) Show the Taylor series of the following functions about $x = 0$ up to the third order.

a) $e^x \cos x$

b) $\log_e (1 + \sin x)$

(2) Evaluate the line integral along C_1

$$\int_{C_1} (x^2 - y)\ ds,$$

where C_1 is the line segment with end points $(0, 0)$ and $(1, 2)$ and s is the arc-length variable of C_1.

(3) Evaluate the line integral along C_2

$$\int_{C_2} (x + 1)\ ds,$$

where C_2 is the parabola given by

$$C_2 :\ y = \frac{1}{2}x^2 \qquad (0 \leq x \leq 1),$$

and s is the arc-length variable of C_2.

2. Solve the following problems.

(1) Find the general solutions of the following ordinary differential equations.

a) $\left(x^2 - y^2\right) \dfrac{dy}{dx} = 2xy$

b) $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = e^{-x} \cos 2x + 2x$

(2) Find the general solution of the following system of ordinary differential equations for y and z.

$$\begin{cases} \dfrac{dy}{dx} - 2\dfrac{dz}{dx} + 3z = 0 \\ \dfrac{dy}{dx} - 4y - 3z = 0 \end{cases}$$

3. The $n \times n$ matrices A and J are given by

$$A = \begin{pmatrix} a_1 & a_2 & a_3 & \cdots & a_n \\ a_n & a_1 & a_2 & \cdots & a_{n-1} \\ a_{n-1} & a_n & a_1 & \cdots & a_{n-2} \\ \vdots & \vdots & \vdots & & \vdots \\ a_2 & a_3 & a_4 & \cdots & a_1 \end{pmatrix}, \qquad J = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix},$$

$(a_i \neq 0,\ i = 1, 2, \cdots, n)$. Solve the following problems, where I is the identity matrix of order n.

(1) Obtain J^2 and J^3 when $n = 4$.

(2) Express $a_1 I + a_2 J + a_3 J^2 + \cdots + a_n J^{n-1}$ using A.

(3) Obtain the determinant $|J - \omega I|$.

(4) Show that the eigenvalues λ_k $(k = 1, 2, \cdots, n)$ of A are given by

$$\lambda_k = a_1 + a_2 \omega_k + a_3 \omega_k^2 + \cdots + a_n \omega_k^{n-1},$$

where ω_k $(k = 1, 2, \cdots, n)$ are the eigenvalues of J.

4. As shown in Fig. 1, $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ are the fundamental vectors of the Cartesian coordinate system (x, y, z), and $\boldsymbol{e}_r$, $\boldsymbol{e}_\theta$, and $\boldsymbol{e}_z$ are the fundamental vectors of the cylindrical coordinate system (r, θ, z). The length of the fundamental vectors is 1. The surface S is defined by

$$S: \ \boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + \sqrt{x^2 + y^2 + 1}\boldsymbol{k} \qquad (x^2 + y^2 \leq R^2)$$

in the Cartesian coordinate system. Solve the following problems, where $\boldsymbol{n}$ is the unit normal vector of the surface S.

(1) Express $\boldsymbol{e}_r$, $\boldsymbol{e}_\theta$, and $\boldsymbol{e}_z$ with $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$, and θ.

(2) Express $\dfrac{\partial \boldsymbol{e}_r}{\partial r}$, $\dfrac{\partial \boldsymbol{e}_r}{\partial \theta}$, $\dfrac{\partial \boldsymbol{e}_\theta}{\partial r}$, $\dfrac{\partial \boldsymbol{e}_\theta}{\partial \theta}$, $\dfrac{\partial \boldsymbol{e}_z}{\partial r}$, and $\dfrac{\partial \boldsymbol{e}_z}{\partial \theta}$ in the cylindrical coordinate system.

(3) Express the surface S in the cylindrical coordinate system.

(4) Obtain $\boldsymbol{n}$ in the cylindrical coordinate system.

(5) The vector field $\boldsymbol{A}$ is given by

$$\boldsymbol{A} = \theta z \boldsymbol{e}_\theta + \sqrt{2r^2 + 1}\boldsymbol{e}_z.$$

Evaluate the surface integral

$$\int_S \boldsymbol{A} \cdot \boldsymbol{n} dS.$$

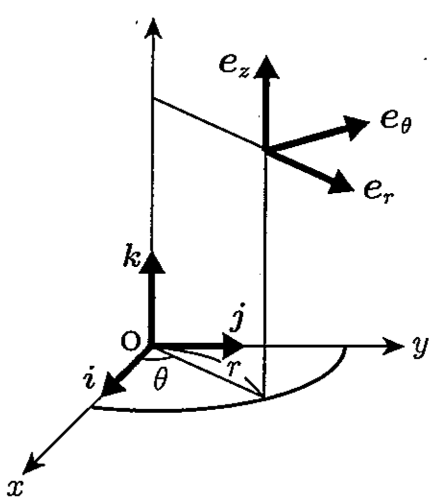

Fig. 1

英

平成 29 年度 秋季募集
(平成 29 年 10 月・平成 30 年 4 月入学)
東北大学大学院機械・知能系入学試験

試験問題冊子

数学B　MATHEMATICS B

平成 29 年 8 月 29 日(火)
Tuesday, August 29, 2017　13:30 − 15:00

Notice

1. Do not open this test booklet until instructed to do so.
2. A test booklet, answer sheets, draft sheets, and a selected-problems form are provided. Put your examinee number on each of the answer sheets, the draft sheets, and the form.
3. Select two of the three problems, and answer them. Indicate your selection on the selected-problems form. Use two answer sheets for each problem.
4. At the end of the examination, double-check your examinee number and the problem numbers on the answer sheets. Put your answer sheets in numerical order on top of the other sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

数学 B　MATHEMATICS B

1. The complex Fourier series of a real function $f(x)$ is defined by

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx},$$

where

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\, e^{-inx}\, dx.$$

Solve the following problems.

(1) The Fourier series of the function $f(x)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx).$$

Express a_n and b_n using c_n and c_{-n}.

(2) Show

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{imx}\, e^{-inx}\, dx = \begin{cases} 1 & (m = n) \\ 0 & (m \neq n) \end{cases},$$

where m and n are integers.

(3) Obtain the Fourier series of $f(x) = \sum_{k=1}^{4} \cos^k x$.

2. The Laplace transform of a function $f(t)$ is defined by

$$\mathcal{L}[f(t)] = F(s) = \int_0^\infty f(t)\, e^{-st}\, dt.$$

Solve the following problems.

(1) Derive the Laplace transform of the n-th derivative of $f(t)$, where n is a positive integer.

(2) Obtain the Laplace transforms of $\sin at$ and $\cos at$ using the result of problem (1), where a is a positive constant.

(3) Derive

$$\mathcal{L}\left[\int_0^t f(t-\tau)\, g(\tau)\, d\tau\right] = F(s)G(s),$$

where $G(s)$ is the Laplace transform of the function $g(t)$.

(4) Solve

$$x(t) - \int_0^t \sin 2(t-\tau)\, x(\tau)\, d\tau = \sin 2t + \cos 2t$$

using the Laplace transform.

3. The function $u(x, y)$ satisfies the partial differential equation

$$\frac{\partial^2 u}{\partial x \partial y} - \frac{\partial^2 u}{\partial y^2} = e^{x+3y}.$$

Consider the coordinate transformation $\xi = x + sy$ and $\eta = x + ty$, where s and t are real numbers ($s \neq t$). Solve the following problems.

(1) Express $\dfrac{\partial^2 u}{\partial x^2}$, $\dfrac{\partial^2 u}{\partial y^2}$, and $\dfrac{\partial^2 u}{\partial x \partial y}$ using u, ξ, and η.

(2) Obtain the partial differential equation for u with respect to ξ and η.

(3) Obtain the general solution $u(x, y)$ of problem (2) when $s = 0$ and $t = 1$.

英

平成 29 年度 秋季募集
(平成 29 年 10 月入学・平成 30 年 4 月入学)
東北大学大学院機械・知能系入学試験

試験問題冊子
【専門科目　Specialized Subjects】

熱力学	THERMODYNAMICS
流体力学	FLUID DYNAMICS
材料力学	STRENGTH OF MATERIALS
機械力学	DYNAMICS OF MECHANICAL SYSTEMS
制御工学	CONTROL ENGINEERING
材料物性学	MATERIALS SCIENCE
電磁気学	ELECTROMAGNETICS
量子力学	QUANTUM MECHANICS

平成 29 年 8 月 30 日 (水)　9:00 － 12:00
Wednesday, August 30, 2017　9:00 － 12:00

Notice

1. Do not open this test booklet until instructed to do so.
2. A test booklet, answer sheets, draft sheets, and two selected-subjects forms are provided. Put your examinee number on each of the answer sheets, the draft sheets, and the form.
3. Select two subjects from the eight subjects in the booklet and answer all problems in each subject. Indicate your selection on the selected-subjects form. Use one set of two answer sheets for each subject, and use one sheet per problem.
4. At the end of the examination, double-check your examinee number and the problem numbers on the answer sheets. Put your answer sheets in numerical order on top of the other sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

熱 力 学　THERMODYNAMICS

1. Consider water at the triple point. Specific volumes of water in solid, liquid and gas phases are expressed as v_s, v_l and v_g, respectively. Specific entropies of water in solid, liquid and gas phases are expressed as s_s, s_l and s_g, respectively. Heat of vaporization, heat of fusion and heat of sublimation of water are expressed as r_V, r_f and r_s, respectively. Temperature at the triple point is expressed as T_t. Answer the following questions.

(1) Express r_V, r_f and r_s using T_t, s_s, s_l and s_g.

(2) Show that the following equation is satisfied.

$$r_s = r_V + r_f$$

(3) Derive the Clapeyron–Clausius equation using the equation of the specific Gibbs free energy.

(4) Show that the slope of the sublimation curve $\left(\frac{dp}{dT}\right)_s$ is larger than that of the vaporization curve $\left(\frac{dp}{dT}\right)_v$ near the triple point on the pressure–temperature $(p-T)$ diagram. Assume that the following equations are satisfied.

$$v_g >> v_s$$
$$v_g >> v_l$$
$$r_s > r_V$$

熱 力 学 THERMODYNAMICS

2. Consider a quasi-static process that 1 kg of an ideal gas changes from state 1 to state 2 in a steady flow system. The kinetic and the potential energies can be neglected. The pressure, specific volume, temperature, specific energy and specific enthalpy of the gas at the state 1 are p_1, v_1, T_1, u_1 and h_1, respectively. Those at the state 2 are p_2, v_2, T_2, u_2 and h_2, respectively. The gas constant is R. Answer the following questions when $p_1 > p_2$ and $v_1 < v_2$.

   (1) Express h_1 using u_1, p_1 and v_1.
   (2) Let q_{12} be the heat transferred to the gas during the process. Obtain the absolute work and the technical work (mechanical work in a steady flow system). Show the areas corresponding to the absolute work and the technical work in the pressure–specific volume ($p-v$) diagram.
   (3) When the process is isothermal, show that the amount of the absolute work is equal to that of the technical work.
   (4) When the process is adiabatic, show which of the absolute work and the technical work is larger. Explain the reason referring to the flow work.

流 体 力 学 FLUID DYNAMICS

1. Consider a two-dimensional irrotational flow of inviscid and incompressible fluid in the xy plane. The x and y components of the velocity of this flow, u and v, are given by

$$u = ax^2 + by^2 + c \quad \text{and} \quad v = dxy \quad (a, b, c, d : \text{constants}) .$$

Answer the following questions.

(1) Find the relationship among the constants, a, b, c and d, when the continuity equation is satisfied.

(2) Find the relationship among the constants, a, b, c and d, when the irrotational flow condition is satisfied.

(3) Obtain the velocity potential $\phi(x, y)$ and the stream function $\psi(x, y)$ of this flow using x, y and c, when $\phi(0,0) = \psi(0,0) = 0$, and $d = 2$.

2. As shown in Fig. 1, water with the density ρ is ejected horizontally from a hole on a side wall of a tank with wheels. As a result of the water ejection, the tank moves with constant acceleration α on a horizontal surface. The gravitational acceleration g is directed vertically downwards, and the water surface is inclined at an angle θ to the horizontal plane. The cross-sectional area of the hole is A, and the hole is located at the depth of H below the water surface along the right side wall. The water is ejected at a constant velocity U relative to the hole. Since the tank is sufficiently large, it is assumed that the changes in a total mass of the tank and water inside of the tank M, the shape and the height of the water surface are negligibly small. Besides, the pressure around the tank is atmospheric, and the viscosity of water and any friction exerted on the tank during the horizontal movement are neglected. Answer the following questions.

(1) Express the momentum of the ejecting water from the hole per unit time, using ρ , A and U, in the reference frame which moves with the tank.
(2) Express the acceleration α using ρ , A, U and M.
(3) Express the angle θ using α and g.
(4) Obtain the ejection velocity U.

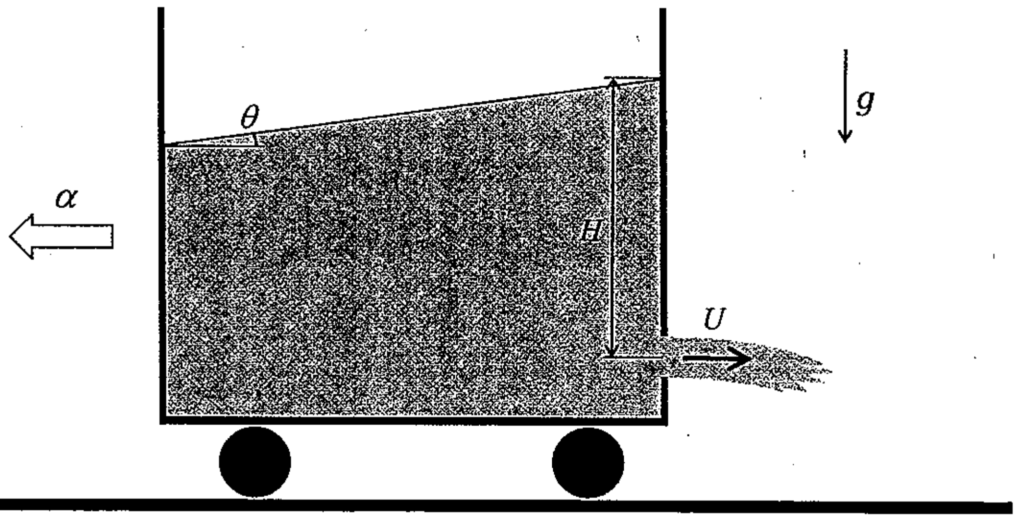

Fig. 1

材 料 力 学　STRENGTH OF MATERIALS

1. Consider a stepped circular shaft ABCD as shown in Fig. 1. Portion AB is a hollow circular shaft with outer diameter $2d$, inner diameter d, and length L. Portion BC is a solid circular shaft with diameter $2d$ and length L. Portion CD is a solid circular shaft with diameter d and length L. The modulus of elasticity in shear of this stepped circular shaft is G. Answer the following questions.

   (1) As shown in Fig. 1(a), the shaft is fixed to a rigid wall at end A. When the shaft is subjected to a twisting moment M_t at position D, determine the specific angles of twist of portions AB, BC, and CD, and the angle of twist at position D.

   (2) As shown in Fig. 1(b), the shaft is fixed to rigid walls at both ends A and D. When the shaft is subjected to a twisting moment M_t at position C, determine the twisting moments M_{tA} and M_{tD} at ends A and D, and the angle of twist at position C.

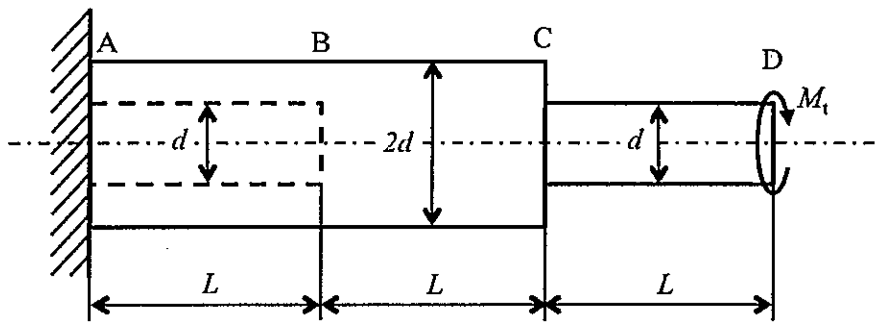

Fig. 1(a)

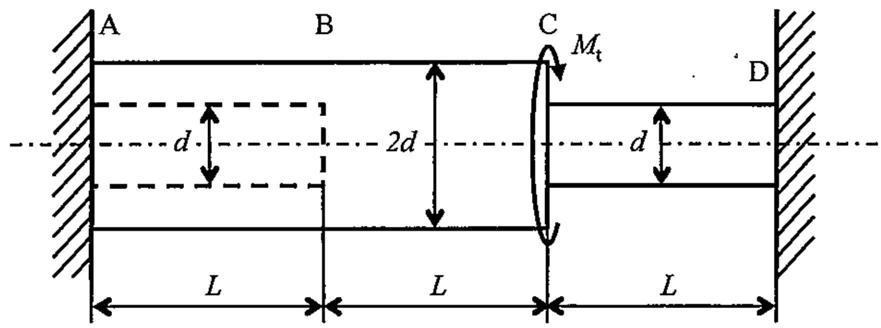

Fig. 1(b)

2. A straight beam AB of length L is subjected to a downward vertical concentrated load W, as shown in Fig. 2. The lengths of AC and BC are a and b, respectively. The flexural rigidity of the beam is EI, and the weight of the beam is negligible. Answer the following questions.

(1) The beam is simply supported at both ends A and B, as shown in Fig. 2(a). Determine the deflection y_c of the beam at position C.

(2) The beam is held at both ends A and B by two springs AD and BE of the same initial length, fixed onto a rigid ceiling vertically, as shown in Fig. 2(b). Each spring has the same spring constant k, and the weight of the springs is negligible. When the elongations of the springs are small enough, determine the vertical displacement δ_c at position C.

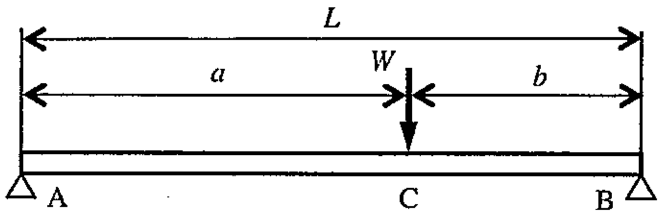

Fig. 2(a)

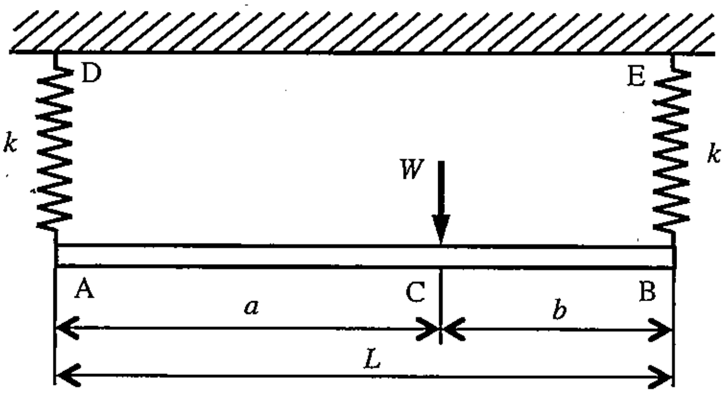

Fig. 2(b)

機 械 力 学 DYNAMICS OF MECHANICAL SYSTEMS

1. Consider a system in a plane, which consists of two springs and two uniform rigid bars AB and CD as shown in Fig. 1. The rigid bars are supported by pins at points E and D, and points B and C are connected by a massless rigid link. The masses and lengths of the two rigid bars are m and L, respectively, and the spring constant of both springs is k. The rigid bars rotationally vibrate with sufficiently small displacements around their equilibrium positions. The angular displacement of the rigid bar AB from the equilibrium position at time t is denoted by $\theta(t)$. Assuming that the masses of the springs and the horizontal deformation are negligible, answer the following questions.

   (1) Obtain the moment of inertia J_{AB} of the rigid bar AB about point E.
   (2) Express the angular displacement $\varphi(t)$ of the rigid bar CD from the equilibrium position by $\theta(t)$.
   (3) Obtain the kinetic energy of the system.
   (4) Obtain the potential energy of the system.
   (5) Derive the equation of motion of the system.

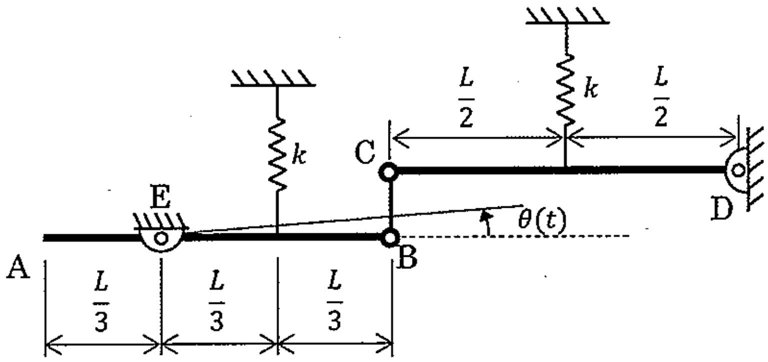

Fig. 1

2. Consider a system which consists of three trailers and five springs as shown in Fig. 2. Trailer 1 with mass m_1 is placed on a horizontal floor, and is connected to the wall by a spring with spring constant k_1. Trailer 2 with mass m_2 is placed on Trailer 1, and is connected to Trailer 1 by two springs with spring constant k_2. Trailer 3 with mass m_3 is placed on Trailer 2, and is connected to Trailer 2 by two springs with spring constant k_3, while being oscillated laterally by an external force $F(t) = P \sin \omega t$, where P is the amplitude of the external force, ω is angular frequency, and t is time. The trailers move only in the horizontal direction with no friction. The absolute displacements of Trailers 1, 2 and 3 from the equilibrium position O are denoted by x_1, x_2 and x_3, respectively. Masses of the springs are negligible. When the system is in a steady state, answer the following questions.

(1) Derive the equation of motion for each trailer.

(2) When $m_1 = m_2 = 2m$, $m_3 = m$, $k_1 = k$, and $k_2 = k_3 = k/2$, find all the natural angular frequencies of the system.

(3) Under the condition of question (2), determine ω at which x_3 becomes zero. In addition, find the amplitude ratio of x_1 to x_2 at each of the determined angular frequencies.

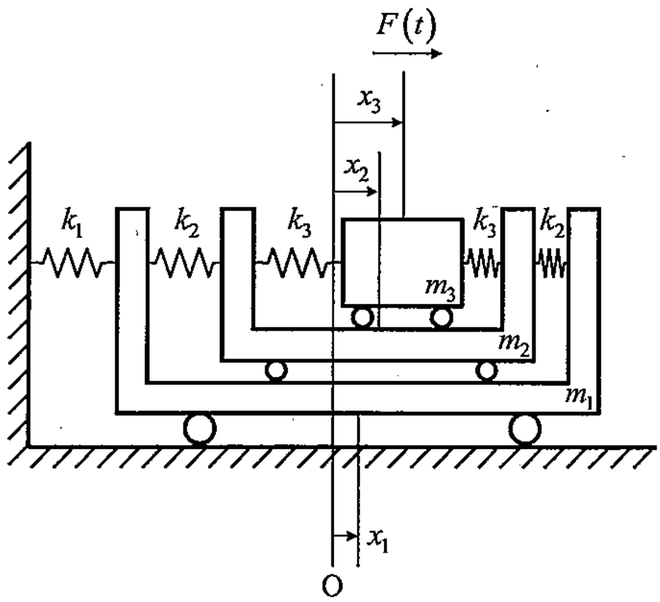

Fig. 2

制御工学 CONTROL ENGINEERING

1. Consider the feedback system shown in Fig. 1, where

$$G(s) = \frac{k}{s(2s+1)(s+4)},$$

and k is a positive constant. Solve the following problems.

(1) Show the loop transfer function of this system and draw the outline of its vector locus.

(2) The phase crossover frequency is defined as the angular frequency at which the phase angle of a frequency response of the loop transfer function is $-180°$. Obtain the phase crossover frequency of this system.

(3) Derive the value of k for the stability limit of this system.

(4) Derive the transfer function from input $U(s)$ to output $Y(s)$ of this system and find the condition of k to stabilize the system.

(5) Find the condition of k so that the steady-state velocity error e_v of the system shown in Fig. 1 satisfies $e_v \leq 0.4$.

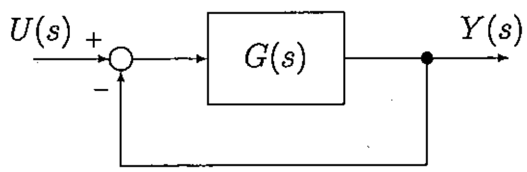

Fig. 1

2. Solve the following problems.

(1) Consider the system expressed by

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t),$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix},$$

where $u(t)$ is input, $y(t)$ is output and $\boldsymbol{x}(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T$ is a state vector. Examine the controllability and the observability of this system.

(2) Obtain the transfer function from the input to the output of the system in problem (1).

(3) Consider a coordinate transformation $\boldsymbol{x}(t) = \boldsymbol{T}\boldsymbol{z}(t)$ that transforms the system in problem (1) into the controllable canonical form

$$\begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_1 & -a_2 \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t),$$

$$y(t) = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix},$$

where $\boldsymbol{z}(t) = \begin{bmatrix} z_1(t) & z_2(t) \end{bmatrix}^T$. Determine the transformation matrix $\boldsymbol{T}$ and the constants a_1, a_2, c_1 and c_2.

(4) Draw a block diagram that represents the system in problem (3). Indicate u, y, z_1 and z_2 in the diagram.

(5) Suppose that a state feedback $u(t) = -\begin{bmatrix} k_1 & k_2 \end{bmatrix} \boldsymbol{x}(t)$ is applied to the system in problem (1). Express the feedback gains k_1 and k_2 in terms of the constants α and β when the characteristic equation of the closed-loop system is $s^2 + \alpha s + \beta = 0$.

(6) In the closed-loop system under the state feedback control given by problem (5), find the condition of k_1 and k_2 so that the system is asymptotically stable.

材 料 物 性 学　MATERIALS SCIENCE

1. When stress is applied to metal, multiple dislocations are generated. Answer the following questions about the dislocation multiplication mechanism. Let be b the size of Burgers vector of dislocation, and G the shear modulus of the metal.

(1) Consider dislocation segment AB intersected with other dislocations and pinned at two ends on its slip plane at points A and B as shown in Fig.1. The dislocation will be multiplied due to application of shear stress τ on the slip plane. Give the name of the dislocation multiplication mechanism.

(2) Consider the dislocation segment AB pinned at the two ends bows out by application of the shear stress τ as shown in Fig.2. The force f applied to the dislocation is given as $f=\tau bl$. Obtain the tension T working on the dislocation segment, where l is the length of the dislocation segment without stress and θ is an angle of the bowing as shown in Fig.2.

(3) Energy of a dislocation per unit length E_d is given as $E_d=Gb^2$. Obtain the relation between τ and the dislocation segment bowing angle θ, and obtain the maximum shear stress τ_{max} .

(4) Illustrate how dislocation is multiplied when the shear stress exceeds τ_{max}.

(5) Explain the role of the multiplication of dislocation in the plastic deformation of metals.

(6) For randomly distributed dislocations, the length of dislocation l is expressed $l=\rho_d^{-1/2}$ where ρ_d is the dislocation density. Obtain the yield stress k as a function of ρ_d.

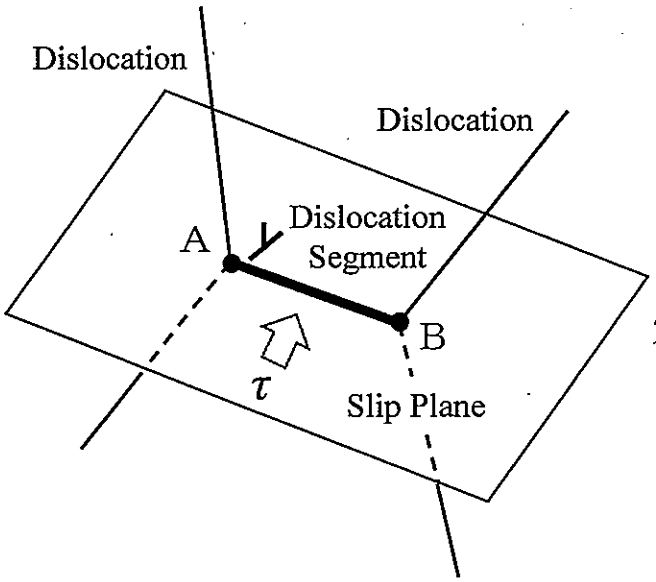

Fig.1 Dislocation segment AB intersected with other dislocations and pinned at two ends on its slip plane at point A and B.

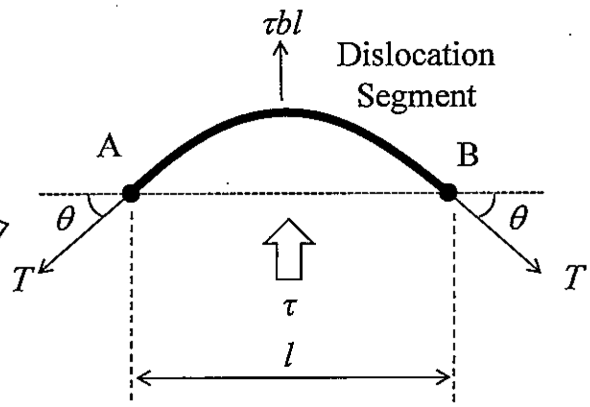

Fig.2 Dislocation segment AB pinned at two ends bows by application of the shear stress (τ) on the slip plane.

2. One side of an iron plate is exposed to carburizing environment at the temperature of 973 K, and the other side is exposed to decarburizing environment at the same temperature. It is assumed that the steady state of carbon diffusion is achieved. The concentration of carbon at the depth of 5.0×10^{-3} and 1.0×10^{-2} m from the carburizing environment are 1.2 and 0.8 kg/m^3, respectively. The carbon concentration just beneath the decarburizing surface is zero. The diffusion coefficient of carbon in iron at temperature of 973 K is 3.0×10^{-11} m^2/s. Answer the following questions. Use the universal gas constant $R = 8.31\,J/(mol\cdot\mathrm{K})$, e = 2.72, ln2 = 0.69, and ln10 = 2.30, if necessary.

(1) Obtain the carbon flux through the iron plate.
(2) Obtain the carbon concentration at the depth of 1.0×10^{-3} m from the surface of the carburizing side.
(3) Obtain the thickness of the iron plate.
(4) Obtain the activation energy for diffusion of carbon when the frequency factor is 6.0×10^{-7} m^2/s.
(5) Obtain the temperature where the diffusion coefficient is a half of the value at 973 K.

1. A magnetic dipole is a pair of magnetic charges $+q_m$ and $-q_m$ with very small distance s, as shown in Fig. 1. A magnetic dipole moment is defined as $\boldsymbol{m} = q_m s$. Answer the following questions, where μ_0 is the permeability.

(1) Draw the magnetic field lines for the magnetic dipole shown in Fig. 1 and also draw the magnetic field lines for a loop current with radius a and current I shown in Fig. 2.

(2) Magnetic field intensity $\boldsymbol{H}$ can be obtained as $\boldsymbol{H} = -\nabla\phi_m$ using magnetic scalar potential ϕ_m. The magnetic scalar potential ϕ_m generated by magnetic charge q_m with distance R is expressed as

$$\phi_m = \frac{q_m}{4\pi\mu_0 R} \ .$$

Obtain the magnetic field intensity $\boldsymbol{H}$ at P_1 ($z = z_1$, $z_1 >> s$) on the z-axis generated by the magnetic dipole shown in Fig. 1.

(3) Obtain the magnetic flux density $\boldsymbol{B}$ at P_2 ($z = z_2$, $z_2 >> a$) on the z-axis generated by the loop current with radius a and current I shown in Fig. 2.

(4) Show that the amplitude of magnetic dipole moment m is expressed by $m = \mu_0 I \pi a^2$ using the results of questions (2) and (3) assuming that the magnetic field generated by the magnetic dipole is equivalent to that by the loop current.

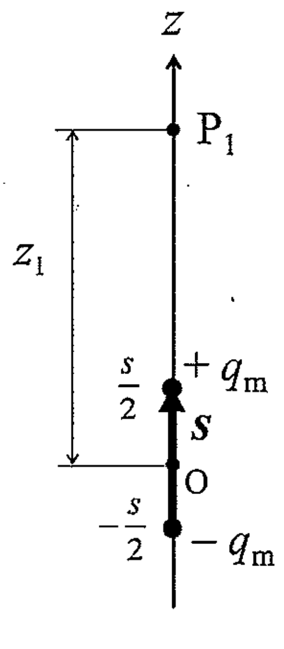

Fig. 1

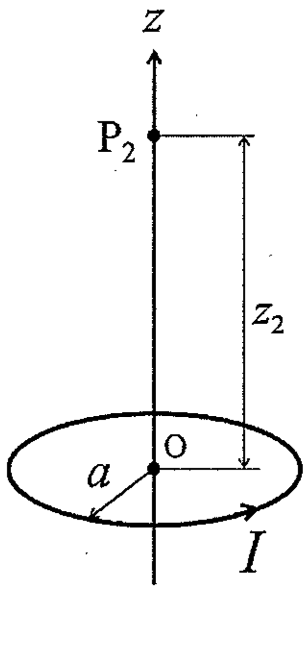

Fig. 2

2. As shown in Fig. 3, two coaxial solenoid coils, C_1 and C_2, have the same length L and the same number of turns N. The radii of coils C_1 and C_2 are r_1 and r_2 ($r_1 > r_2$), respectively. The coil C_1 is connected to an alternative current power source providing a current $I = I_0 \sin(\omega t)$. The coil C_2 is connected to resistor R and a switch which is open in the initial state. Answer the following questions. Use the permeability μ_0 and neglect the edge effect and electrical resistance of the coils.

(1) Derive the corresponding integral expression from the differential relation between the magnetic field intensity $\boldsymbol{H}$ and the current density $\boldsymbol{J}$ without displacement current.

(2) Obtain the corresponding integral expression from the differential relation between the magnetic flux density $\boldsymbol{B}$ and the electric field $\boldsymbol{E}$.

(3) Find the magnetic field intensity in the coil C_1.

(4) Find the power source voltage and then evaluate the supplying power from the source.

(5) Find the voltage between both ends of the coil C_2.

(6) When the switch is closed, how does the voltage between both ends of the coil C_2 change, and how does the supplying power obtained in question (4) change?

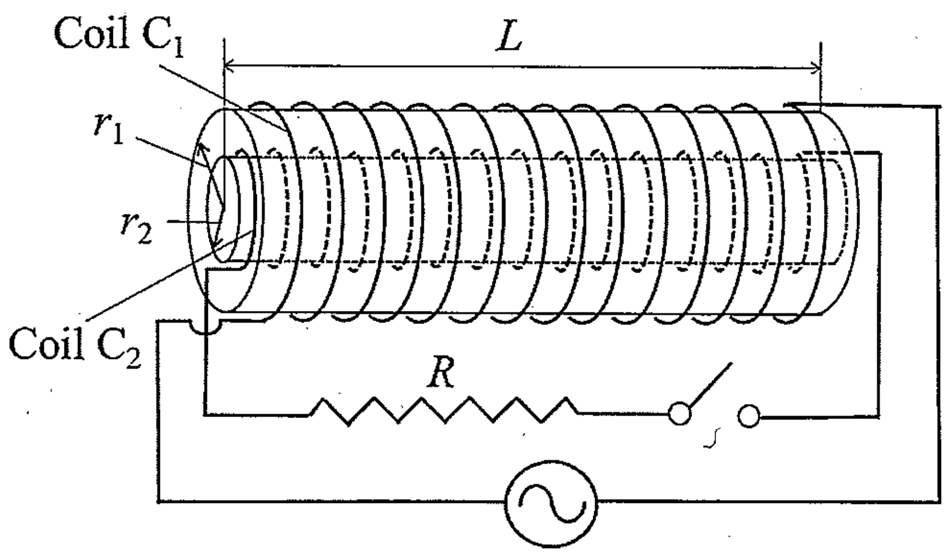

Fig. 3

量 子 力 学　QUANTUM MECHANICS

1. Consider the wave function $\varphi(r,\theta)$ of a free particle of mass m and energy E. The particle is moving along a circular orbit of the radius r in a two-dimensional space whose center is located at the origin. The Laplacian in polar coordinates is given by $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$. Answer the following questions.

   (1) Show the two-dimensional time-independent Schrödinger equation for the particle in polar coordinates (r,θ).

   (2) Show the periodic boundary condition for $\varphi(r,\theta)$.

   (3) Derive the normalized wave function $\varphi(r,\theta)$.

   (4) Find the energy of the particle E.

   (5) Show that it is impossible to precisely determine the location of the particle for a given angular momentum.

2. Consider photoelectrons emitted from a metal plate irradiated by monochromatic light with the light quantity P [W] and the wavelength λ [m]. The energy of each photoelectron is E [J]. Answer the following questions. Use the speed of light c [m/s] in vacuum, Planck's constant h [J·s] and the work function of the metal plate w [J].

(1) Express w using c, h, E and λ.

(2) Express the number of photons per unit time of the monochromatic light using P, c, h and λ.

(3) Find the amount of change in the energy of each photoelectron when the wavelength changes from λ to $\lambda/2$.

(4) Find the amount of change in the energy of each photoelectron when the light quantity increases from P to $2P$.