

## Reference Problems - STRENGTH OF MATERIALS

1. There are three columns whose cross-sectional area is  $A_0$  and whose natural length at temperature  $T_0$  is  $L_0$ . These three columns are bonded to the rigid plates at the top and bottom ends, as shown in Fig. 1. The column located in the center of the rigid plate has a longitudinal modulus of elasticity of  $E_1$  and a linear expansion coefficient of  $\alpha_1$ , while the two columns located at distance  $C$  on both sides of the central column have a longitudinal modulus of elasticity of  $E_2$  and a linear expansion coefficient of  $\alpha_2$ . Here,  $0 < E_1 < E_2$  and  $0 < \alpha_1 < \alpha_2$ . Ignore the self-weight of the columns and rigid plates. Answer the following questions.

- (1) At temperature  $T_0$ , an object of weight  $W_0$  is placed on the center of the upper rigid plate, as shown in Fig. 2. Find the strain in the columns. Also, find the stress acting on each column.
- (2) After the object of weight  $W_0$  is removed from the state in the question (1), the temperature is increased by  $\Delta T$  from  $T_0$ . Find the strain in the columns. Also, find the stress acting on each column.
- (3) The object of weight  $W_0$  is placed on the center of the upper rigid plate as in the question (1), and the temperature is increased by  $\Delta T$  from  $T_0$ . Find the stress acting on each column.

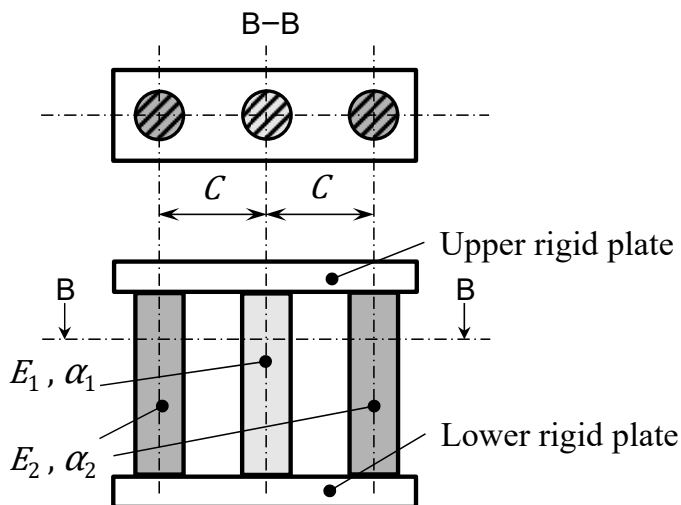


Fig. 1

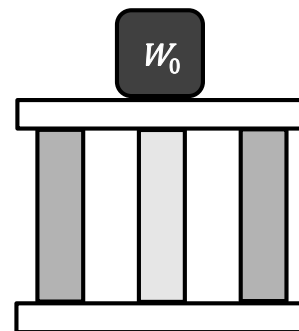


Fig. 2

2. Answer the following questions.

- (1) The plane stresses ( $\sigma_x=200$  MPa,  $\sigma_y=0$  MPa,  $\tau_{xy}=100$  MPa) are given as shown in Fig. 3. Draw the Mohr's stress circle corresponding to this stress state. Find the magnitude and direction of the maximum and minimum principal stresses.
- (2) The yield strength of a homogeneous isotropic material was evaluated by a uniaxial tensile test, and the material yielded when the tensile stress reached 250 MPa. Determine whether or not this material will yield when subjected to the plane stress as shown in Fig. 3, and the reason should also be given. Here, the yield will occur when the maximum shear stress reaches the material-specific limit value.

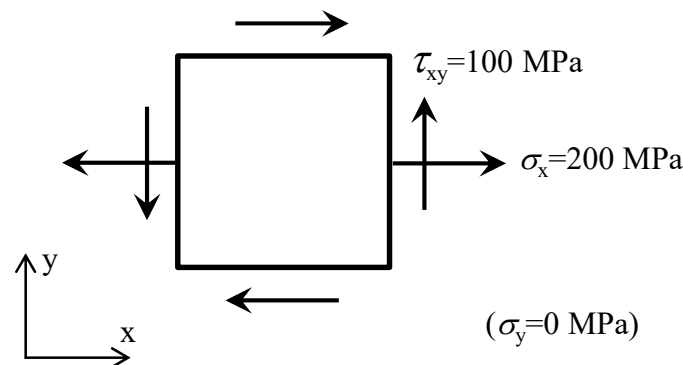


Fig. 3

3. Consider a stepped solid circular shaft shown in Fig. 4. The diameter and length of section AB are  $d_1$  and  $L/3$ , and those of section BD are  $d_2$  and  $2L/3$ , respectively.  $d_1$  is smaller than  $d_2$ . The shaft is fixed at the both ends, A and D, to vertical rigid walls. The shear modulus of the shaft material is  $G$ . As shown in Fig. 1, the shaft is twisted by the torsional moment  $M_t$  at position C, midpoint of section BD. Answer the following questions. Describe also the processes of deriving the answers. Ignore stress concentration.

- (1) Find the torsional moments developed at the ends of the shaft, position A and

position D, respectively.

- (2) Find the angle of twist  $\phi_c$  at position C.
- (3) Determine the ratio of torsional stress acting on section AB and torsional stress acting on section CD.

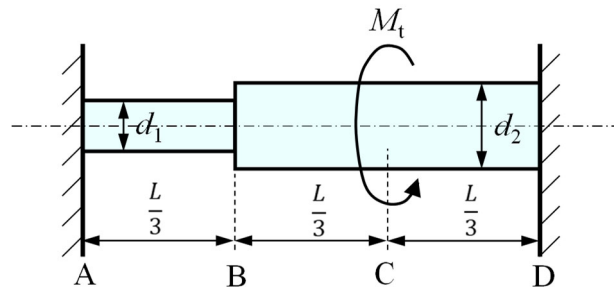


Fig. 4

4. A  $3L$ -long beam is supported at two points A and B at a distance  $L$  from both ends of the beam, as shown in Fig. 5. A concentrated load  $W_1$  is applied at the left end of the beam and a concentrated load  $W_2$  is applied at the right end, where  $2W_2 > W_1 > W_2$ . Answer the following questions. Neglect the weight of the beam. Describe also the processes of deriving the answers.

- (1) Determine the reaction forces acting at position A and position B, respectively.
- (2) Draw the shear force diagram.
- (3) Draw the bending moment diagram.

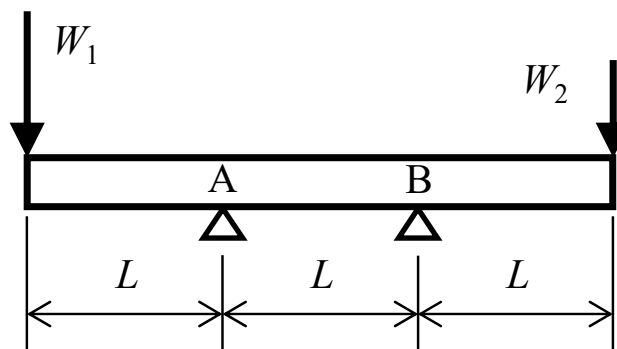


Fig. 5

